VCAA Mathematical Methods Exam 1 Solutions 2005
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Part I

1	2	3	4	5	6	7	8	9
D	**B**	**D**	**E**	**D**	**D**	**E**	**D**	**E**

10	11	12	13	14	15	16	17	18
A	**D**	**A**	**A**	**C**	**E**	**A**	**B**	**C**

19	20	21	22	23	24	25	26	27
C	**C**	**D**	**C**	**D**	**B**	**E**	**B**	**A**

Q1 $\Pr(X \ge 2) = \frac{7}{30} + \frac{10}{30} + \frac{4}{30} = \frac{21}{30}$ D

Q2 $Mean = 0\times\frac{3}{30} + 1\times\frac{6}{30} + 2\times\frac{7}{30} + 3\times\frac{10}{30} + 4\times\frac{4}{30} = \frac{66}{30}$ B

Q3 $\Pr(Z < z) = 0.95$, $z = 1.645$ D

Q4 $\Pr(X \ge 1) = 1 - \Pr(X = 0) = 1 - \frac{{}^2C_0 \times {}^4C_2}{{}^6C_2}$

$= 1 - \frac{1\times 6}{15} = \frac{9}{15}$ E

Q5 Binomial: $p = 0.3,\ q = 0.7,\ x = 0$

$\Pr(X = 0) = {}^nC_0 (0.3)^0 (0.7)^n = 0.0576,\ \therefore 0.7^n = 0.0576,$

$n = \frac{\log_e 0.0576}{\log_e 0.7} = 8$. D

Q6 Factorise $x^4 + 3x^3 - 4x^2 - 12x = x(x^3 + 3x^2 - 4x - 12)$

$= x((x^3 + 3x^2) - (4x - 12)) = x(x^2(x+3) - 4(x-3))$

$= x(x+3)(x^2 - 4) = x(x+3)(x-2)(x+2)$ D

Q7 f has a turning point at $x = 3$, $a \ge 3$ E

Q8 $e^{2x} = \frac{4}{3},\ \therefore 2x = \log_e\left(\frac{4}{3}\right),\ \therefore x = 0.144.$ D

Q9 $5\log_{10} x - 2\log_{10} x = 3,\ 3\log_{10} x = 3,$

$\log_{10} x = 1,\ \therefore x = 10.$ E

Q10 Between $x = 0$ and $x = 1$, there are $2\frac{1}{2}$ periods.

$\therefore \frac{5T}{2} = 1,\ T = \frac{2}{5}.\ \therefore \frac{2\pi}{n} = \frac{2}{5},\ n = 5\pi.$ Amplitude = 2 and translated upwards by 1 unit. A

Q11 $n = \frac{1}{4}$. For $\tan nx$, the period is $\frac{\pi}{n},\ \therefore T = 4\pi.$ D

Q12 $\cos(2x) = \sqrt{3}\sin(2x)$, $0 < 2x < \pi$.

$\therefore \tan(2x) = \frac{1}{\sqrt{3}},\ 2x = \frac{\pi}{6},\ \therefore x = \frac{\pi}{12}.$ A

Q13 For $f(x) < 0$, the graph of $a\sin(x)$ needs to be shifted downwards by a distance greater than the amplitude a, $\therefore -c > a$, i.e. $c < -a$. A

Q14 Reflection in the line $y = x$. Vertical asymptote: $x = 1$. C

Q15 $y = x^3$, a horizontal translation of -2, $y = (x+2)^3$, then a dilation (factor ½) from the y-axis, $y = (2x+2)^3$. E

Q16 x-intercepts at $x = -1,\ x = 3$ and $x = 1$. The last one is a turning point. A

Q17 Vertical asymptote gives $b = -1$, horizontal asymptote gives $c = 2.\ \therefore y = \frac{a}{(x-1)^2} + 2.$ Use $(0,0)$ to find a.

$\therefore 0 = \frac{a}{(-1)^2} + 2.\ \therefore a = -2.$ B

Q18 $y = \frac{x-2}{x+3} = \frac{x+3-5}{x+3} = \frac{x+3}{x+3} - \frac{5}{x+3} = 1 - \frac{5}{x+3}$.
Vertical asymptote: $x = -3$; horizontal asymptote: $y = 1$. C

Q19 $f'(x)$ is always positive. As $x \to 0^+$, $f'(x) \to +\infty$.
As $x \to +\infty$, $f'(x) \to 0^+$. C

Q20 At $x = 0$, $f(0) = 1$. At $x = 1$, $f(1) = 1 + e$.
Average rate $= \frac{f(1) - f(0)}{1 - 0} = \frac{1 + e - 1}{1} = e.$ C

Q21 $\frac{d}{dp}\left(10p(1-p)^9\right) = 10(1-p)^9 - 9(10p)(1-p)^8$

$= 10(1-p)^8((1-p) - 9p) = 10(1-p)^8(1 - 10p)$ D

Q22 Let $u = e^{2x}$ and $y = f(e^{2x}) = f(u)$.

The chain rule: $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = f'(u)\times(2e^{2x})$

$= 2e^{2x} f'(e^{2x})$. C

Q23 At $x = 2,\ y = 0$, and $m = \frac{dy}{dx} = 8x^3 - 12x^2 = 16.$
Equation of tangent: $y = 16(x-2)$. D

Q24 The curve has positive gradient from $-\infty$ to 2 exclusive of 0 and 2. B

Q25 $f'(x) = 2\cos(2x) - \sin(x)$. Sketch $y = 2\cos(2x) - \sin(x)$ and $y = -0.8$ to find 4 intersections in $[0, 2\pi]$. E

Q26 $y = \int 3(2x-1)^{-\frac{3}{2}} dx = \frac{3(2x-1)^{-\frac{1}{2}}}{-\frac{1}{2}\times 2} + c = \frac{-3}{(2x-1)^{\frac{1}{2}}} + c$ B

Q27 Note that $\int_0^0 f(t)dt = 0$, $\int_x^0 f(t)dt < 0$, where $-a \le x < 0$, because $f(t) < 0$ and $\int_0^x f(t)dt > 0$, where $0 < x \le a$, because $f(t) > 0$.

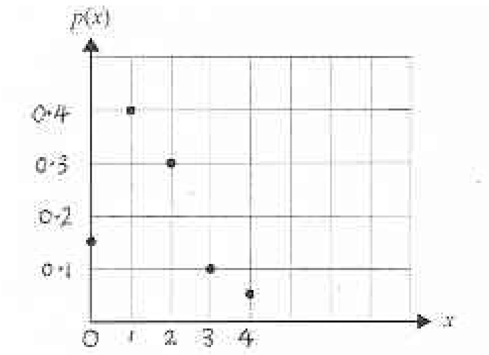
$\therefore$ for $x \ne 0$, $0 < x \le a$, $G(x) > 0$.

Also for $-a \le x < 0$, $G(x) = \int_0^x f(t)dt = -\int_x^0 f(t)dt > 0$. A

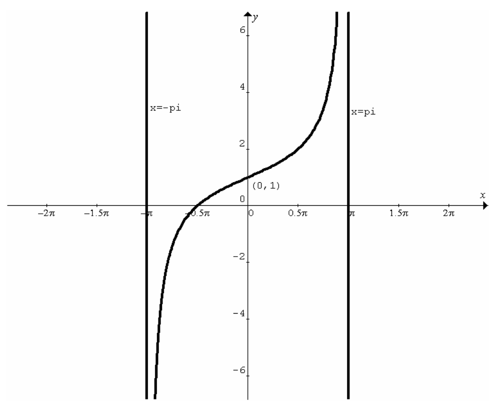
Part II

Q1 $\Pr(X < 46) = normalcdf(-E99,46,41,3) = 0.952$.

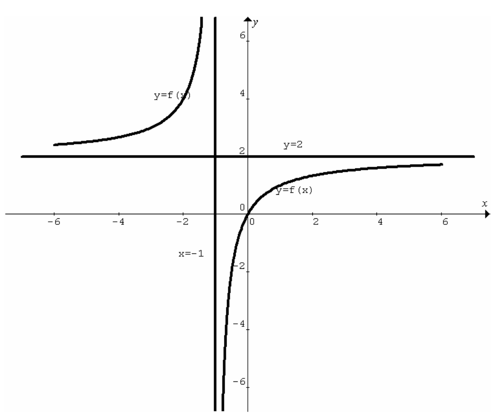
Q2



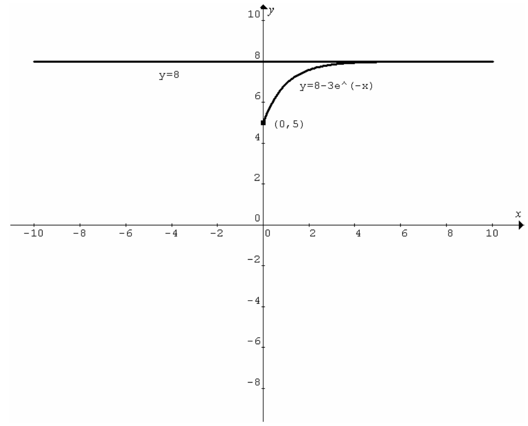
Q3



Q4



Q5a



Q5b Equation of *f*: $y = 8 - 3e^{-x}$, equation of f^{-1}: $x = 8 - 3e^{-y}$.

Re-express with *y* as the subject: $e^{-y} = \frac{8-x}{3}$,

$y = -\log_e\left(\frac{8-x}{3}\right)$, $\therefore y = \log_e\left(\frac{3}{8-x}\right)$. The domain of f^{-1} is $[5,8)$.

$\therefore f^{-1} : [5,8) \to R, f^{-1}(x) = \log_e\left(\frac{3}{8-x}\right)$.

Q6a $y = (x+2)(x^2 + bx + c) = x^3 - 2x^2 - 5x + 6$,

$\therefore 2c = 6$, i.e. $c = 3$ and

$2b + c = -5$, $\therefore 2b = -8$, i.e. $b = -4$.

$\therefore y = (x+2)(x^2 - 4x + 3)$.

Q6b $y = (x+2)(x-1)(x-3)$. The *x*-intercepts are:

$(-2,0)$, $(1,0)$ and $(3,0)$.

Q6c Area $= \int_{-2}^{1}(x^3 - 2x^2 - 5x + 6)dx - \int_{1}^{3}(x^3 - 2x^2 - 5x + 6)dx$

$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x\right]_{-2}^{1} - \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x\right]_{1}^{3}$

$= 21.08$ square units

Q7a $e^{kx} = 3^x$, $\therefore kx = \log_e 3^x$, $\therefore kx = x\log_e 3$,

$kx - x\log_e 3 = 0$, $x(k - \log_e 3) = 0$, $\therefore k = \log_e 3$ for all $x \in R$.

Q7b Since $3^x = e^{kx}$, where $k = \log_e 3$, $\therefore y = e^{kx}$.

$\frac{dy}{dx} = ke^{kx} = (\log_e 3)e^{kx} = 3^x \log_e 3$.

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