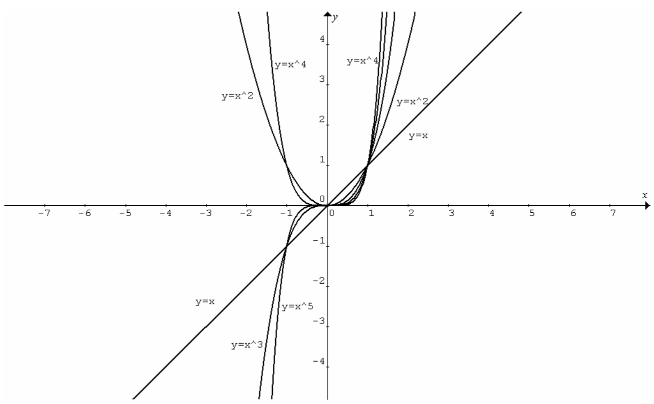
## Functions and graphs

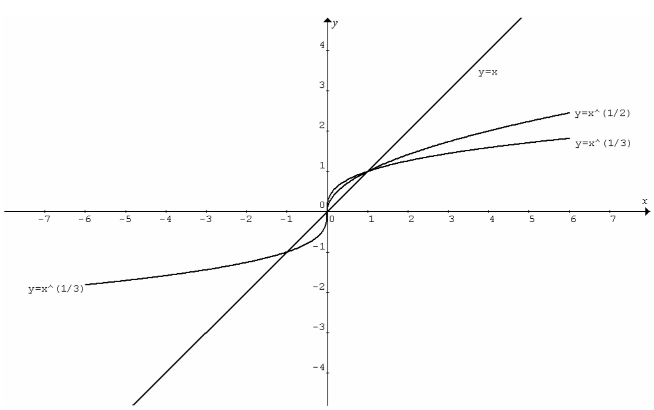
**Power functions** $y = x^n$, for $n \in Q$ (set of rational numbers)

The graph of $y = x$ is a straight line through the origin $(0,0)$. Domain: R; range: R.
The graphs of even-power functions, e.g. $y = x^2$ and $y = x^4$ have a turning-point at $(0,0)$. They show symmetry under reflection in the y-axis. $\therefore$ the y-axis, i.e. the line $x = 0$ is called the axis of symmetry. Domain: R; range: $[0,\infty)$.
The graphs of odd-power functions, e.g. $y = x^3$ and $y = x^5$ have a stationary point of inflection at $(0,0)$. Domain: R; range: R.



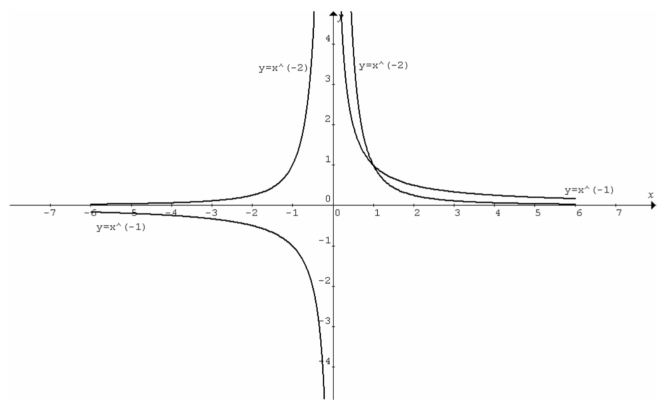

The function $y = x^{\frac{1}{2}}$ can be written as $y = \sqrt{x}$. It is undefined for $x < 0$. It has an end point at $(0,0)$. Domain: $[0,\infty)$; range: $[0,\infty)$.

The function $y = x^{\frac{1}{3}}$ can be expressed as $y = \sqrt[3]{x}$. It has a vertical tangent at $(0,0)$. Domain: R; range: R.



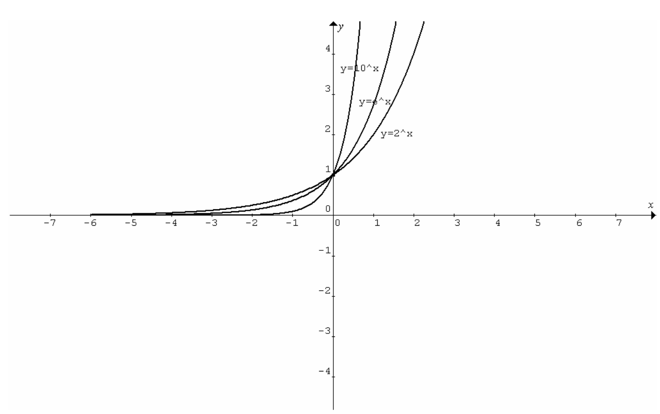

The graph of $y = x^{-1}$ (or $y = \frac{1}{x}$) consists of two branches, one in the first quadrant and the other in the third quadrant. The axis of symmetry is the line $y = x$. The function shows the following asymptotic behaviours:
As $x \to -\infty$, $y \to 0^-$; as $x \to +\infty$, $y \to 0^+$. $\therefore y = 0$ is the horizontal asymptote of the function.
As $x \to 0^-$, $y \to -\infty$; as $x \to 0^+$, $y \to +\infty$. $\therefore x = 0$ is the vertical asymptote.
Domain: R\{0}; range: R\{0}.

The graph of $y = x^{-2}$ (or $y = \frac{1}{x^2}$) also has two branches, one in the first quadrant and the other in the second quadrant. The line $x = 0$ is the axis of symmetry. The function shows the following asymptotic behaviours:
As $x \to -\infty$, $y \to 0^+$; as $x \to +\infty$, $y \to 0^+$. $\therefore y = 0$ is the horizontal asymptote of the function.
As $x \to 0^-$, $y \to +\infty$; as $x \to 0^+$, $y \to +\infty$. $\therefore x = 0$ is the vertical asymptote.
Domain: R\{0}; range: $R^+$.

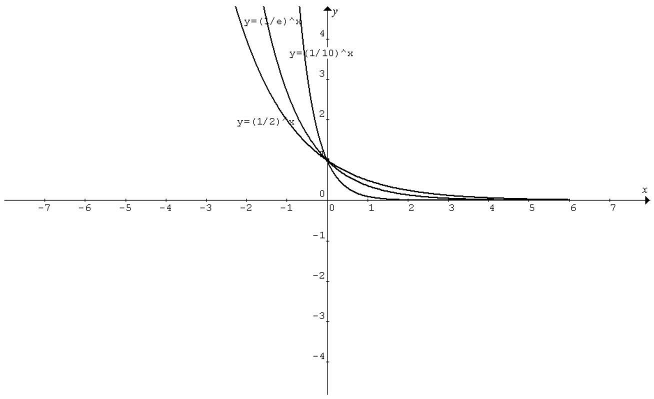



**Exponential functions** $y = a^x$, where $a \in R^+$

For $a > 1$, the graphs of $y = a^x$ have the same shape and the same y-intercept $(0,1)$. Asymptotic behaviour: As $x \to -\infty$, $y \to 0^+$, the same horizontal asymptote $y = 0$ for the functions. $a^x$ is always >0. Domain: R; range: $R^+$.

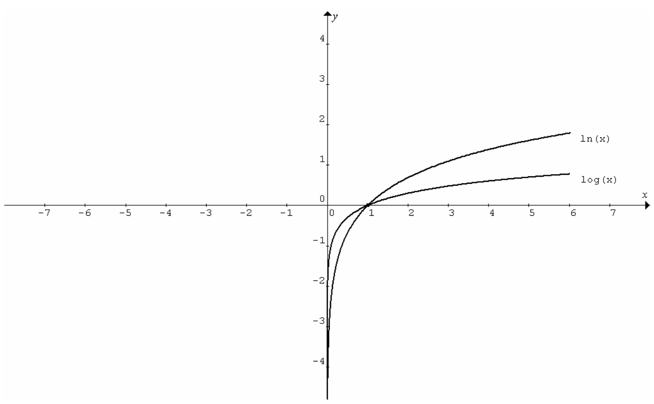


For $0 < a < 1$, the graphs of $y = a^x$ have the same shape and the same y-intercept $(0,1)$. Asymptotic behaviour: As $x \to +\infty$, $y \to 0^+$, the same horizontal asymptote $y = 0$ for the functions. $a^x$ is always >0. Domain: R; range: $R^+$.




**Logarithmic functions** $y = \log_e x$ **and** $y = \log_{10} x$

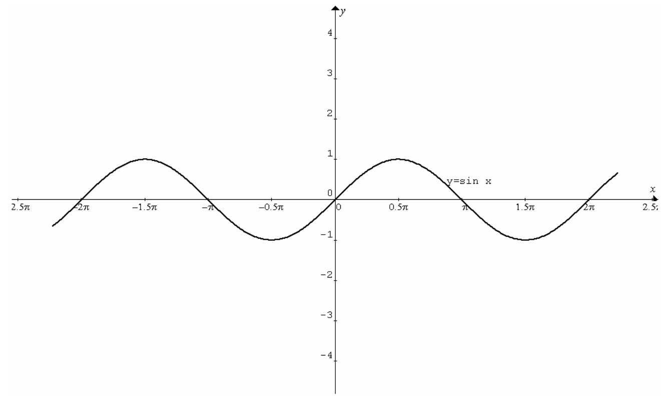
The log functions $y = \log_e x$ (denoted as $\ln(x)$) and $y = \log_{10} x$ (denoted as $\log(x)$) have the same shape and a common x-intercept $(1,0)$. They are undefined for $x \le 0$. Both functions have negative value for $0 < x < 1$ and positive value for $x > 1$. Asymptotic behaviour: As $x \to 0^+$, $y \to -\infty$, the same vertical asymptote $x = 0$. Domain: $R^+$; range: R.

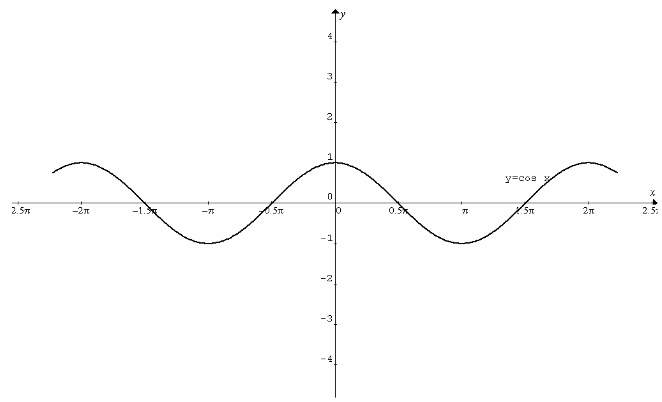



**Circular (trigonometric) functions** $y = \sin x$, $y = \cos x$

Both functions are periodic functions. Each repeats itself after a period $T = 2\pi$, i.e. each has symmetry property under a horizontal translation of $2\pi$, e.g. $\sin(a \pm 2\pi) = \sin a$, $\cos(a \pm 2\pi) = \cos a$. Other symmetry properties: For $\cos x$, under reflection in the y-axis, $\cos(-a) = \cos a$. For $\sin x$, under reflection in the y-axis and a horizontal translation of $\pi$, $\sin(\pi - a) = \sin a$; under reflections in both x and y-axes, $-\sin(-a) = \sin a$.

The value of each function fluctuates between −1 and 1 inclusively, the amplitude of each is 1. Domain: R; range: $[-1,1]$.




**Circular (trigonometric) function** $y = \tan x$

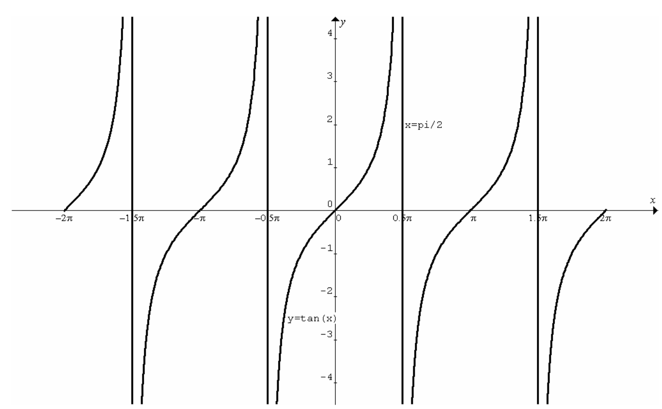
The function $y = \tan x$ is also a periodic function. It repeats itself after a period $T = \pi$, i.e. it has symmetry property under a horizontal translation of $\pi$, $\tan(a \pm \pi) = \tan a$.

It also has symmetry property under reflections in both x and y-axes, $-\tan(-a) = \tan a$.

The term amplitude is not applicable here.

The function is undefined at $x = \pm\left(n - \frac{1}{2}\right)\pi$ for $n \in J^+$ (set of positive integers). It shows asymptotic behaviour as $x \to \pm\left(n - \frac{1}{2}\right)\pi$. The vertical asymptotes are $x = \pm\left(n - \frac{1}{2}\right)\pi$.

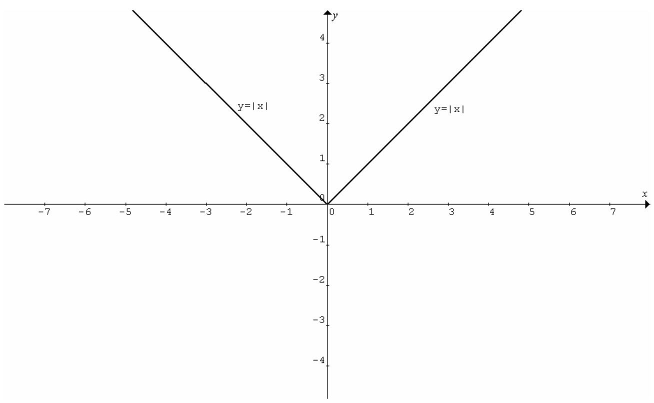
Domain: $\left\{x : x \ne \pm\left(n - \frac{1}{2}\right)\pi, n \in J^+\right\}$; range: R.



**Modulus function** $y=|x|$

$y=|x|$ can be defined as $y=\begin{cases}-x, & x<0 \\ x, & x\geq 0\end{cases}$ or $y=\sqrt{x^2}$.

It has symmetry property under reflection in the y-axis, i.e. $y=|x|$ and $y=|-x|$ are the same, and the line $x=0$ is its axis of symmetry. Its vertex is $(0,0)$.
Domain: R; range: $[0,\infty)$.

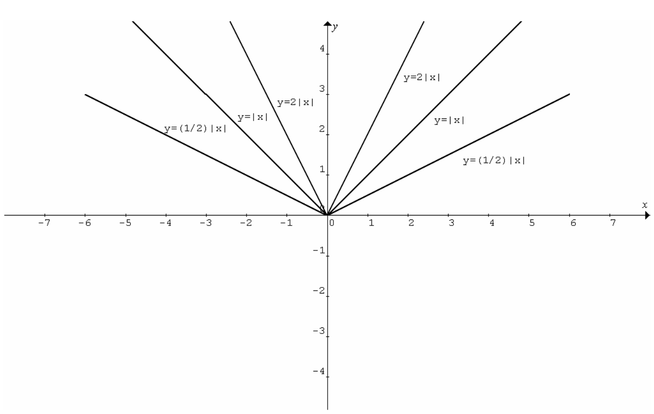



**Transformations**

Any of the above functions can be transformed by one or a combination of the following function operations.

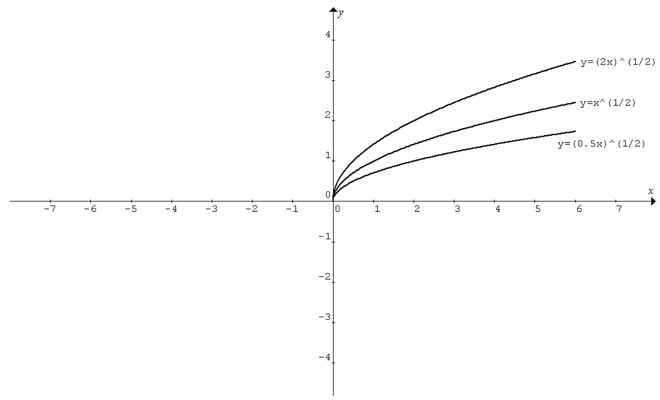
*Vertical dilation of function* $y=f(x)$:
$y=f(x)\rightarrow y=Af(x)$, where $A\in[0,\infty)$. For $0\leq A<1$, the graph of $y=f(x)$ is compressed towards the x-axis to give it a wider appearance; for $A>1$, it is stretched away from the x-axis to give it a narrower appearance. $A$ is called the dilation factor.

Example  Compare the graphs of the transformed functions $y=\frac{1}{2}|x|$ and $y=2|x|$ with the graph of the original function $y=|x|$.



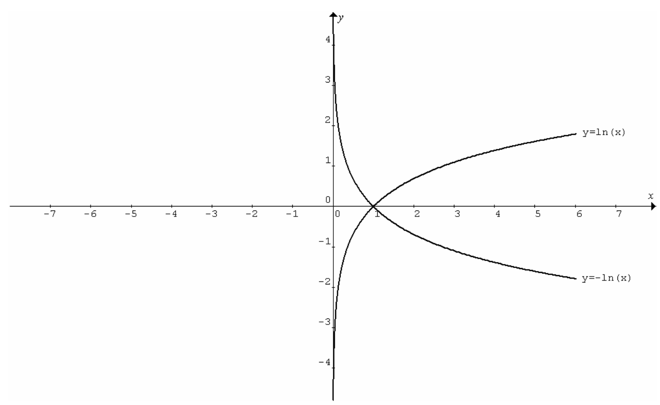

*Horizontal dilation of function* $y=f(x)$:
$y=f(x)\rightarrow y=f(nx)$, where $n>0$. For $0<n<1$, the graph of $y=f(x)$ is stretched away from the y-axis to give it a wider appearance; for $n>1$, it is compressed towards the y-axis to give it a narrower appearance. In this transformation the dilation factor is $\frac{1}{n}$.

Example  Compare $y=\sqrt{0.5x}$ and $y=\sqrt{2x}$ with $y=\sqrt{x}$.



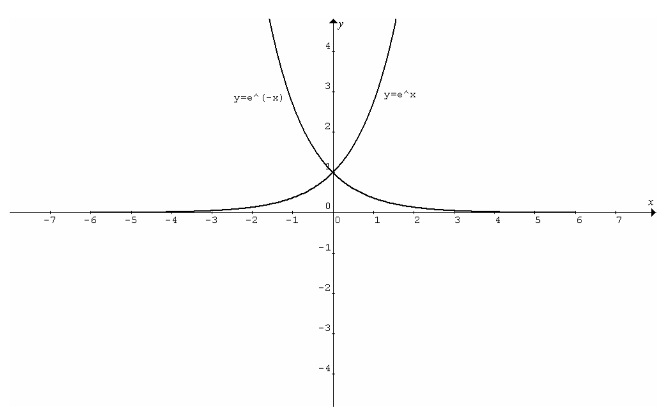

*Reflection of function* $y=f(x)$ *in the x-axis*:
$y=f(x)\rightarrow y=-f(x)$.

Example  Compare $y=-\log_e x$ with $y=\log_e x$.




*Reflection of function* $y=f(x)$ *in the y-axis*:
$y=f(x)\rightarrow y=f(-x)$.

Example  Compare $y=e^{-x}$ with $y=e^{x}$.

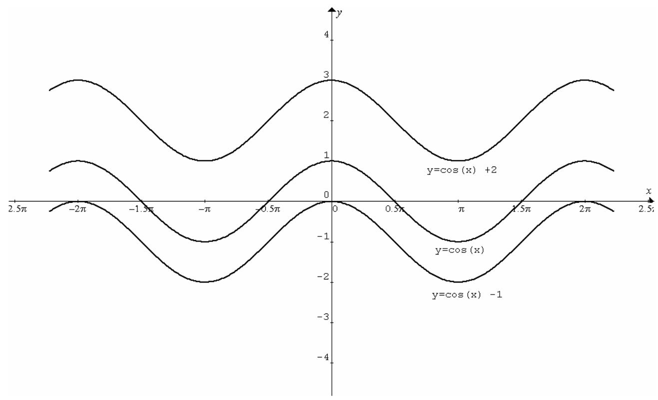


*Vertical translation of function $y = f(x)$ by c units*

$y = f(x) \rightarrow y = f(x) \pm c$, where $c > 0$.

The + and – operations correspond to upward and downward translations respectively.

Example Compare $y = \cos x + 2$ and $y = \cos x - 1$ with $y = \cos x$.

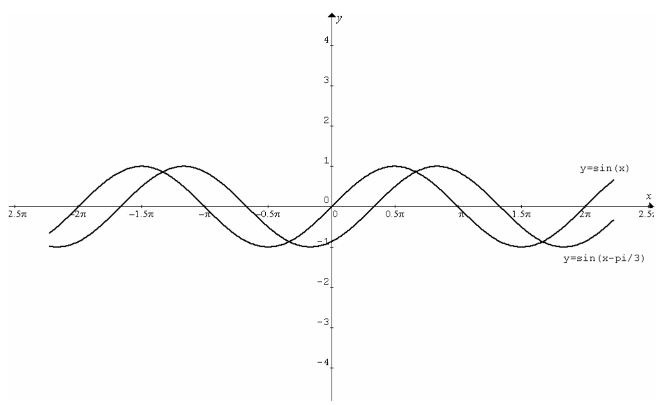



*Horizontal translation of function $y = f(x)$ by b units*

$y = f(x) \rightarrow y = f(x \pm b)$, where $b > 0$.

The + and – operations correspond to left and right translations respectively.

Example Compare $y = \sin\left(x - \frac{\pi}{3}\right)$ with $y = \sin x$.




**Combination of transformations**

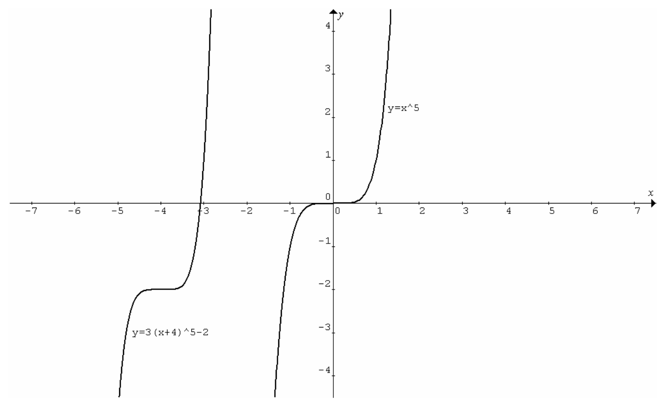
If a transformed function is the result of a combination of the above transformations, it would be easier to recognise the transformations involved by expressing the function in the form $y = \pm A\ f(\pm n(x \pm b)) \pm c$.

Vertical translation (+ up)
Horizontal translation (+ left)
Horizontal dilation (factor 1/*n*)
Reflection in the y-axis (– sign)
Vertical dilation (factor *A*)
Reflection in the x-axis (– sign)

To sketch the transformed function from the original function, always carry out translations last.

Example 1 Sketch $y = 3(x+4)^5 - 2$.

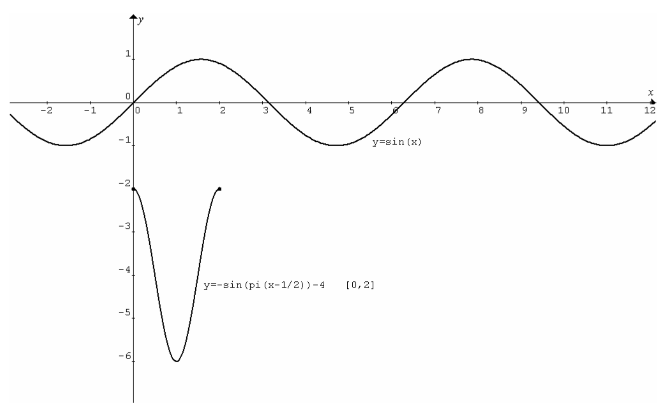
This function is a transformation of $y = x^5$. It involves a vertical dilation by a factor of 3, and translations of 4 left and 2 down. The stationary point of inflection changes from $(0,0)$ to $(-4,-2)$.




Example 2 Sketch $y = -2\sin\left(\pi x - \frac{\pi}{2}\right) - 4$ for $0 \le x \le 2$.

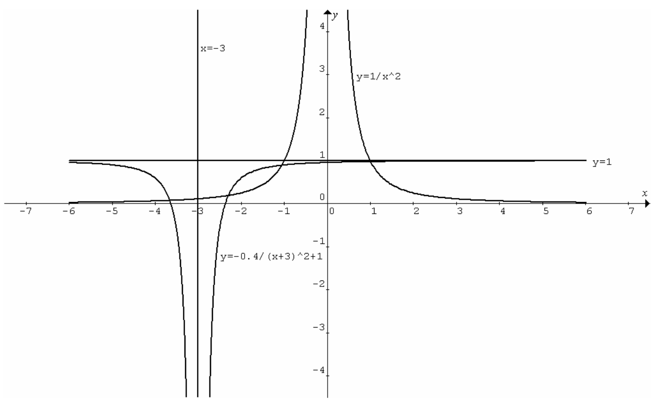
Express the function as $y = -2\sin \pi\left(x - \frac{1}{2}\right) - 4$.

This circular function is a transformation of $y = \sin x$. It has an amplitude of 2 (note: not –2) and a period of $T = \frac{2\pi}{\pi} = 2$. They correspond to a vertical dilation by a factor of 2 and a horizontal dilation by a factor of $\frac{1}{\pi}$ respectively. There is a reflection in the x-axis followed by translations of $\frac{1}{2}$ right and 4 down.



Example 3 Sketch $y = \dfrac{-0.4}{(x+3)^2} + 1$.

This function is the transformation of $y = \dfrac{1}{x^2}$. It involves reflection in the x-axis and a vertical dilation by a factor of 0.4, and then translations 3 left and 1 up. The function has $x = -3$ and $y = 1$ as its asymptotes.




Example 4 Sketch $y = -\left|1 - \dfrac{x}{2}\right| + 1$.

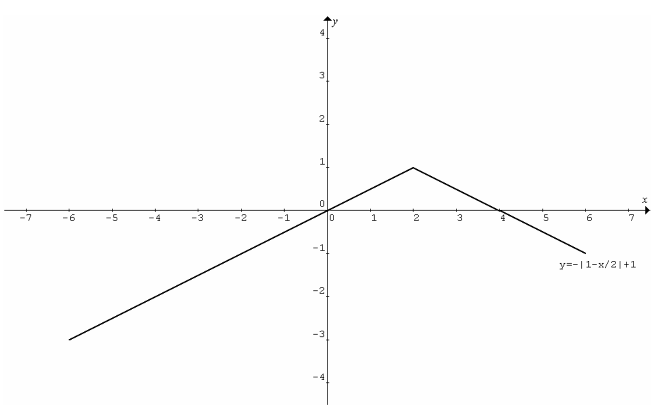
This function is the transformation of $y = |x|$. Firstly express it in the form $y = \pm Af(\pm n(x \pm b)) \pm c$.

$y = -\dfrac{1}{2}|2 - x| + 1 = -\dfrac{1}{2}|-(x-2)| + 1 = -\dfrac{1}{2}|x-2| + 1$. The last step is due to the symmetry property of $y = |x|$ under reflection in the y-axis.

The transformation involves the followings:

Reflection in the x-axis; vertical dilation by factor $\dfrac{1}{2}$, translations 2 right and 1 up.

The vertex is $(2,1)$.



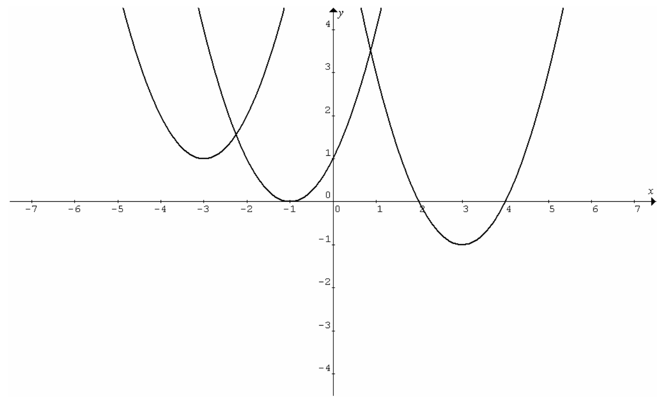

**Polynomial functions**

A polynomial function $P(x)$ is a linear combination of power functions $x^n$, where $n \in \{0,1,2,3,......\}$. Examples are:

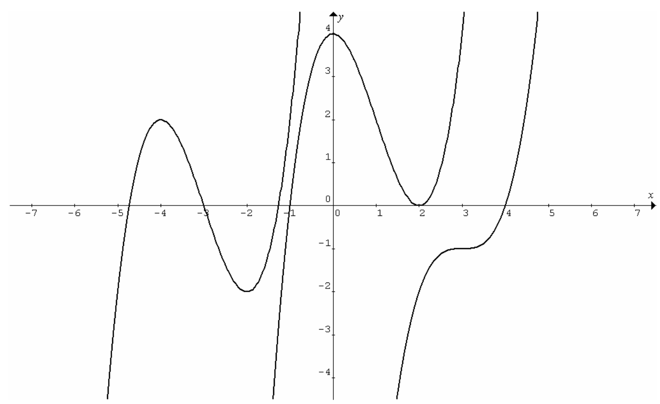
| | |
|---|---|
| $y = 2x - 5$ | Linear function |
| $y = -3x^2 + x + 2$ | Quadratic function |
| $y = 0.2x^3 - \dfrac{x^2}{3} + (\sqrt{5})x - \pi$ | Cubic function |
| $y = -x^4 + (\sqrt[3]{4})x - e^2$ | Quartic function |

Some polynomial functions can be changed to factorised form. The *linear* factors give the x-intercepts. Some polynomial functions may not have any linear factor(s); hence not all polynomial functions have x-intercept(s).

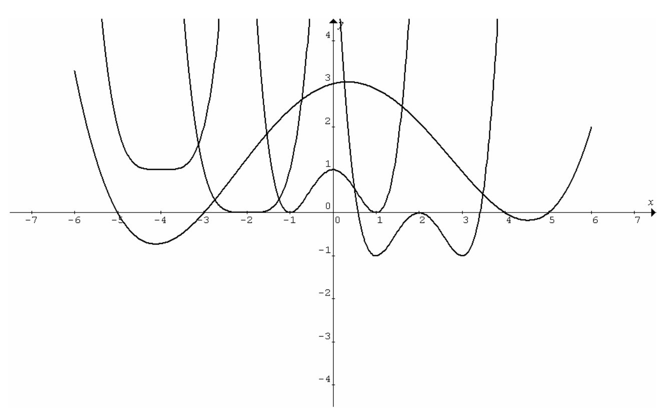
A quadratic function may have 0, 1 or 2 distinct linear factors, hence 0, 1 or 2 x-intercepts.



A cubic function may have 1, 2 or 3 distinct linear factors, hence 1, 2 or 3 x-intercepts.



A quartic function may have 0, 1, 2, 3 or 4 distinct linear factors, hence 0, 1, 2, 3 or 4 x-intercepts.



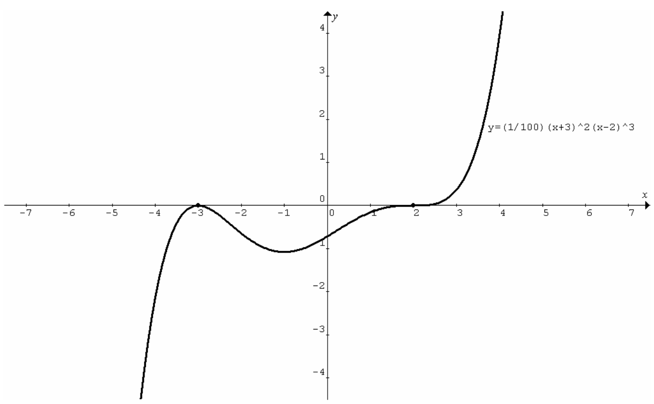
If the power of a linear factor in a polynomial is *even*, then the corresponding x-intercept is a turning point.
If the power of a linear factor in a polynomial is *odd*, then the corresponding x-intercept is a stationary point of inflection.

If $y = a(x-b)^2(x-c)^3(x-d)^4(x^2+e)(x-f)^5$, then the x-intercepts at $x = b$ and $x = d$ are turning points; and the x-intercepts at $x = c$ and $x = f$ are stationary points of inflection. The factor $x^2 + e$ is not linear and $\therefore$ does not correspond to an x-intercept.

If *a* is a positive (negative) value, the graph of a polynomial function heads upwards (downwards) in the positive x-direction.

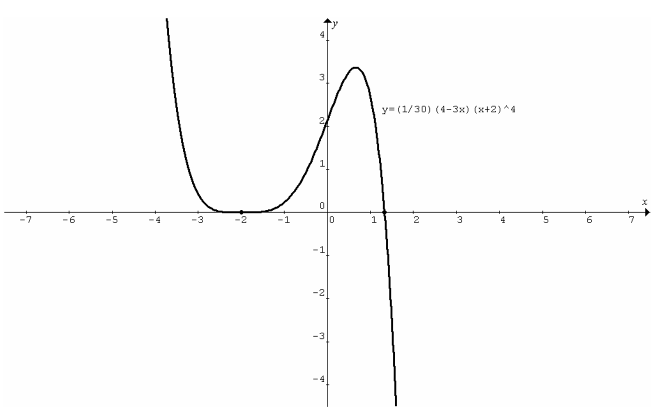
Example 1 Sketch $y = \frac{1}{100}(x+3)^2(x-2)^3$

The x-intercept at $x = -3$ is a turning point; at $x = 2$ the x-intercept is a stationary point of inflection.

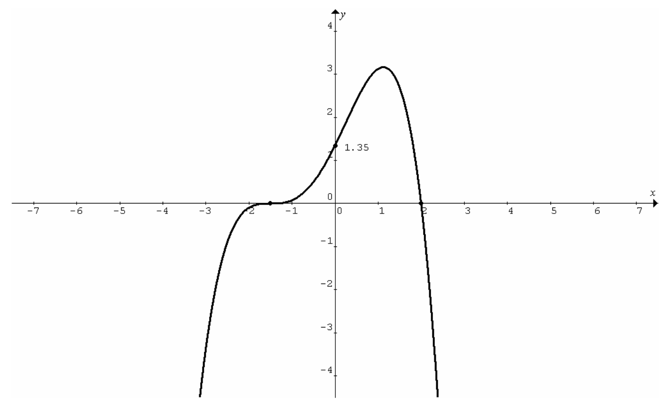



Example 2 Sketch $y = \frac{1}{30}(4-3x)(x+2)^4$

Express the function as $y = -\frac{1}{10}\left(x - \frac{4}{3}\right)(x+2)^4$. The function crosses the x-axis at $x = \frac{4}{3}$; it touches the x-axis at $x = -2$.




Example 3 Find the equation of the quartic function shown below.




At $x = -\frac{3}{2}$ the function has an x-intercept that is a stationary point of inflection; at $x = 2$ the function crosses the x-axis.

Hence, $y = a\left(x + \frac{3}{2}\right)^3(x-2)$, where $|a|$ is the vertical dilation factor to be determined using further information, in this case, the y-intercept $(0, 1.35)$.

$1.35 = a\left(\frac{3}{2}\right)^3(-2)$, $\therefore a = -\frac{1}{5}$, $\therefore y = -\frac{1}{5}\left(x + \frac{3}{2}\right)^3(x-2)$.

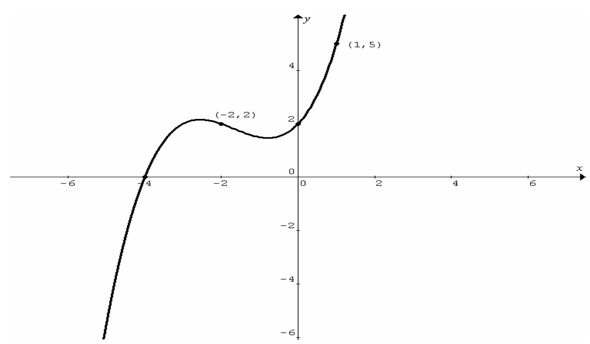
This quartic function can also be expressed as

$y = -\frac{1}{5}\left[\frac{1}{2}(2x+3)\right]^3(x-2) = -\frac{1}{5}\left(\frac{1}{2}\right)^3(2x+3)(x-2)$,

i.e. $y = -\frac{1}{40}(2x+3)(x-2)$.

The – sign corresponds to the observation that the graph headed 'south' in the positive x-direction.

Example 4 Find the equation of the cubic function shown in the graph below.




The cubic function has only one x-intercept at $x = -4$, and $\therefore$ only one linear factor. Its factorised form must be $y = a(x+4)(x^2+bx+c)$. Use the other given points to set up simultaneous equations, then solve for *a*, *b* and *c*.

$(0,2) \rightarrow 2 = 4ac \qquad \therefore ac = 0.5 \qquad (1)$

$(-2,2) \rightarrow 2 = 2a(4-2b+c) \qquad \therefore 4a - 2ab + ac = 1 \qquad (2)$

$(1,5) \rightarrow 5 = 5a(1+b+c) \qquad \therefore a + ab + ac = 1 \qquad (3)$

Substitute eq (1) in eqs (2) and (3),
$4a-2ab=0.5$ (4)
$a+ab=0.5$ (5) $\times 2$
$2a+2ab=1$ (6)
Add eqs (4) and (6), $6a=1.5$, $\therefore a=0.25$ (7)
Sub. eq (7) in (5) to obtain $b=1$.
Sub. eq (7) in (1) to obtain $c=2$.

Hence $y=0.25(x+4)(x^2+x+2)$.

*All* quadratic polynomial functions can be changed to turning point form $y=A(x\pm b)^2\pm c$ by completing the square. The turning point is $(\mp b,\pm c)$.

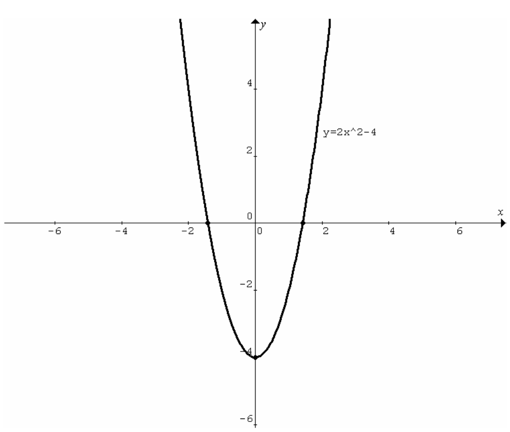
*Some* cubic polynomial functions can be expressed in similar form $y=A(x\pm b)^3\pm c$. $(\mp b,\pm c)$ is the stationary point of inflection of the cubic function.

*Some* quartic polynomial functions can also be expressed in similar form $y=A(x\pm b)^4\pm c$. $(\mp b,\pm c)$ is the turning point of the quartic function.

These forms should be viewed as the transformations (discussed previously) of the power functions, $x^2$, $x^3$ and $x^4$ respectively.

Example 1 Find the turning point and the x-intercepts of $y=2x^2-4$. Sketch its graph.

The function is in turning point form. The turning point is $(0,-4)$.
Factorise $y=2x^2-4=2(x^2-2)=2(x-\sqrt{2})(x+\sqrt{2})$.
The linear factor $x-\sqrt{2}$ gives x-intercept $(\sqrt{2},0)$; the linear factor $x+\sqrt{2}$ gives x-intercept $(-\sqrt{2},0)$.
The y-intercept is obtained by letting $x=0$, $(0,-4)$.



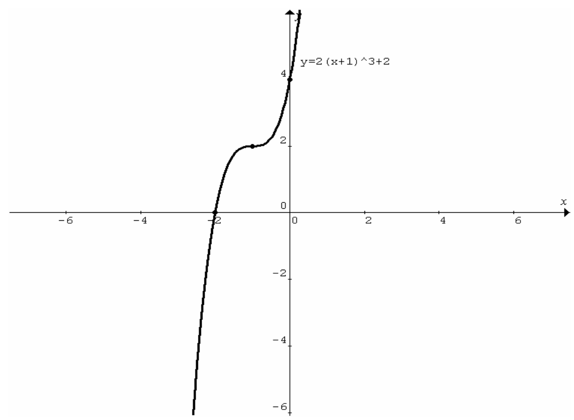

Example 2 Factorise $2(x+1)^3+2$ and then sketch $y=2(x+1)^3+2$.

This cubic function is the sum of two cubes: $y=2(x+1)^3+2$

$=2\left[(x+1)^3+1^3\right]=2[(x+1)+1]\left[(x+1)^2-(x+1)(1)+(1)^2\right]$
$=2(x+2)(x^2+x+1)$.

There is only one linear factor, $\therefore$ only one x-intercept at $x=-2$.
Note that the x-intercept can be obtained by letting $y=0$ and solve for x. $2(x+1)^3+2=0$, $2(x+1)^3=-2$, $(x+1)^3=-1$, $x+1=\sqrt[3]{-1}=-1$, $\therefore x=-2$.

The given function is in stationary inflection point form. The stationary point of inflection is $(-1,2)$.




**Graphs of sum and difference of functions**

The sum (or difference) of two functions *f* and *g* is defined only for $x\in D_f\cap D_g$, where $D_f$ and $D_g$ are the domains of *f* and *g* respectively.

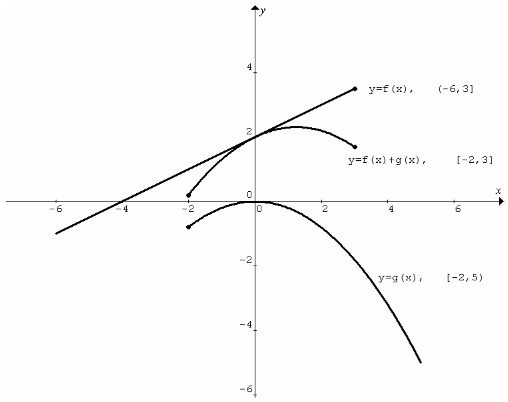
Example Given $y=f(x)=\sqrt{x+1}$ and $y=g(x)=\log_e(2-x)$, find the domain of $f+g$.

$f(x)=\sqrt{x+1}$, $\therefore x+1\geq 0$, $\therefore x\geq -1$, $\therefore D_f=\{x:x\geq -1\}$.

$g(x)=\log_e(2-x)$, $\therefore 2-x>0$, $\therefore x<2$, $\therefore D_g=\{x:x<2\}$.

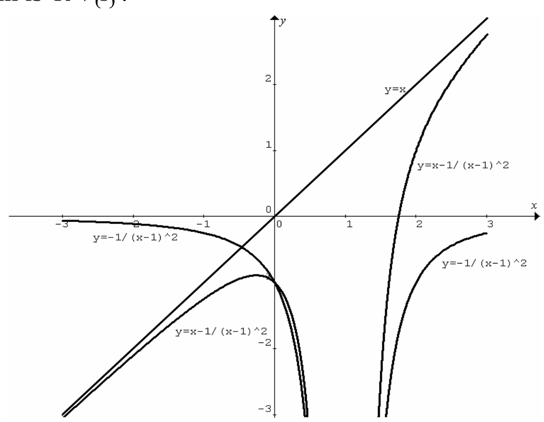
Hence, $D_{f+g}=D_f\cap D_g=\{x:-1\leq x<2\}$, i.e. $[-1,2)$.

If the graphs of $y=f(x)$ and $y=g(x)$ are given, then the graph of $y=f(x)+g(x)$ can be sketched by the method of addition of ordinates (i.e. by adding the y-coordinates of the two functions at several suitable x values in $D_f\cap D_g$).



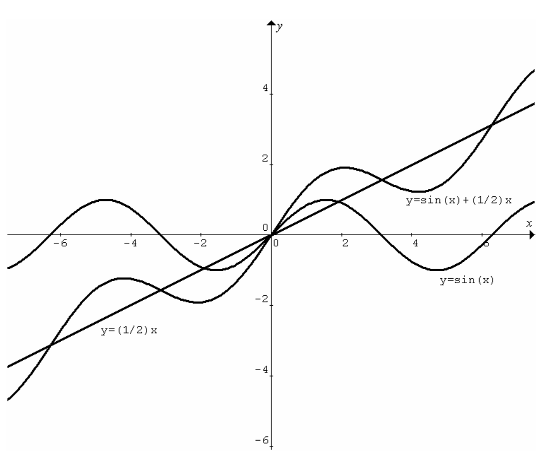

Example 1 Use addition of ordinates to sketch $y = x - \dfrac{1}{(x-1)^2}$.

Sketch $y = x$ and $y = -\dfrac{1}{(x-1)^2}$ on the same axes, then add the y-coordinates of the two functions at several suitable x-values. Note that $y = x - \dfrac{1}{(x-1)^2}$ is undefined at $x = 1$, $\therefore$ its domain is $R \setminus \{1\}$.




Example 2 Sketch $y = \sin x + \dfrac{1}{2}x$ by addition of ordinates.

Sketch $y = \sin x$ and $y = \dfrac{1}{2}x$ on the same axes, then add the y-coordinates of the two functions at several suitable x-values.




**Graph of product of functions**

New functions can be generated by addition (or subtraction) of functions as discussed in the previous section. New functions called products (or quotients) of functions can also be generated by multiplication (or division) of functions.

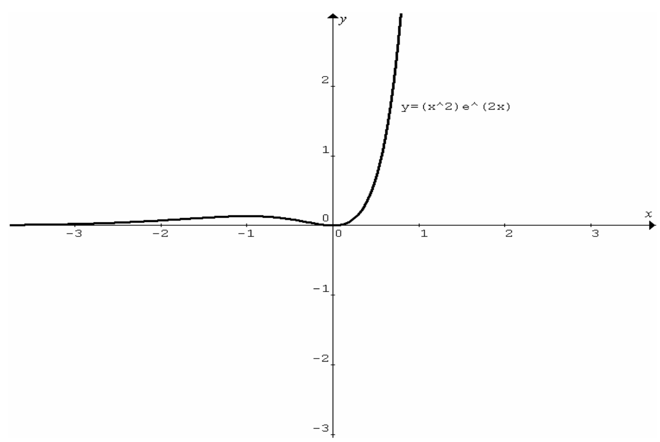
The product (or quotient) of two functions $u$ and $v$ is defined only for $x \in D_u \cap D_v$ and $v \neq 0$ if $v$ is the divisor.

If the graphs of $y = u(x)$ and $y = v(x)$ are given, then the graph of $y = u(x)v(x)$ or $\left( y = \dfrac{u(x)}{v(x)} \right)$ can be sketched by multiplying (or dividing) the y-coordinate of one function by the y-coordinate of the other at several suitable x values within $D_u \cap D_v$.

Example 1

$y = x^2 e^{2x}$, the product of functions $u(x) = x^2$ and $v(x) = e^{2x}$.

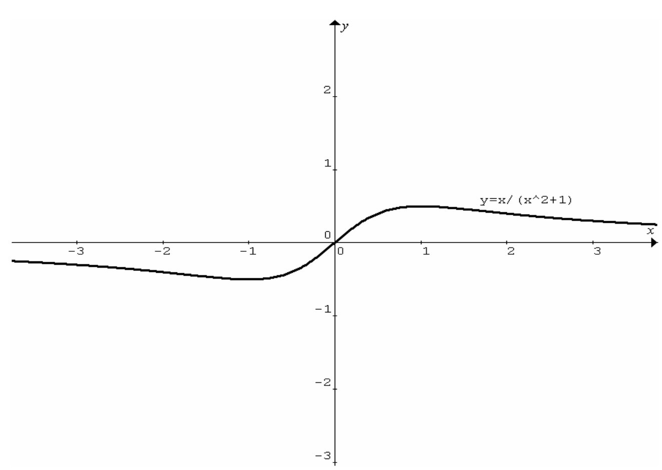
$D_u = R$ and $D_v = R$, $D_{uv} = D_u \cap D_v = R$.




Example 2

$y = \dfrac{x}{x^2 + 1}$, the quotient of functions $u(x) = x$ and $v(x) = x^2 + 1$.

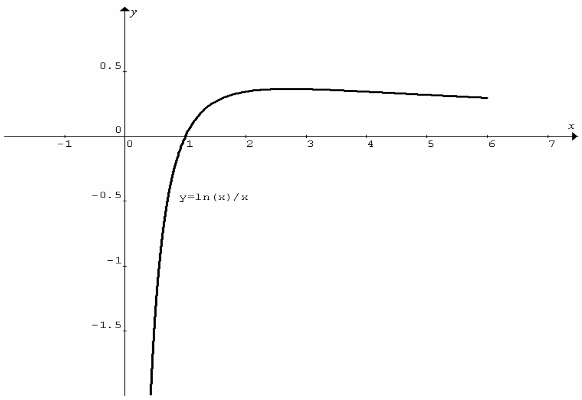
$D_u = R$ and $D_v = R$, $D_{uv} = D_u \cap D_v = R$.



Example 3

$y=\dfrac{\log_e x}{x}$, $u(x)=\log_e x$ and $v(x)=x$.

$D_u=R^+$ and $D_v=R\setminus\{0\}$, $D_{uv}=D_u\cap D_v=R^+$.




**Graphs of composite functions**

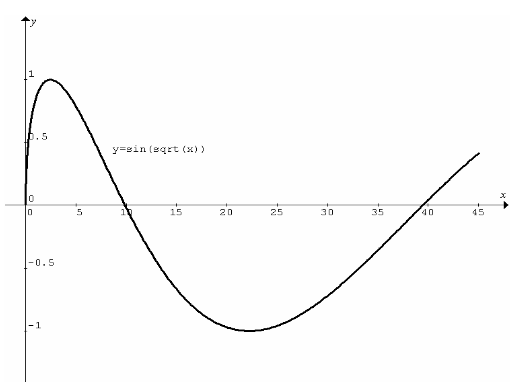
Given two functions $y=f(x)$ and $y=g(x)$, new functions can be generated in the following ways:

In $y=f(x)$ the variable $x$ is replaced by $g(x)$; the new function is $y=f(g(x))$.

In $y=g(x)$ the variable $x$ is replaced by $f(x)$; the new function is $y=g(f(x))$.

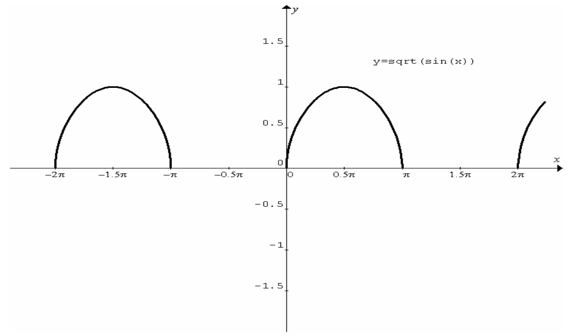
Functions generated in the above manner are called composite functions. The two new composite functions are denoted as $f\circ g$ and $g\circ f$ respectively,
i.e. $f\circ g(x)=f(g(x))$ and $g\circ f(x)=g(f(x))$.

Example 1 Given $f(x)=\sin x$ and $g(x)=\sqrt{x}$, generate two composite functions.
Replacing $x$ by $\sqrt{x}$ in function $f(x)=\sin x$ to obtain $f\circ g(x)=\sin(\sqrt{x})$.




$f\circ g(x)=\sin(\sqrt{x})$ is defined when $\sqrt{x}\in R$ AND $x\ge 0$, i.e. $x\ge 0$. Hence $D_{f\circ g}=\{x:x\ge 0\}$.

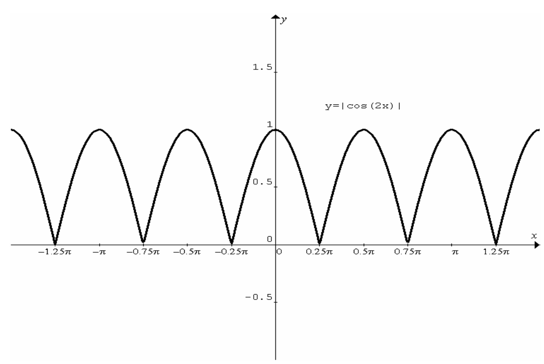
Replacing $x$ by $\sin x$ in function $g(x)=\sqrt{x}$ to obtain $g\circ f(x)=\sqrt{\sin x}$.

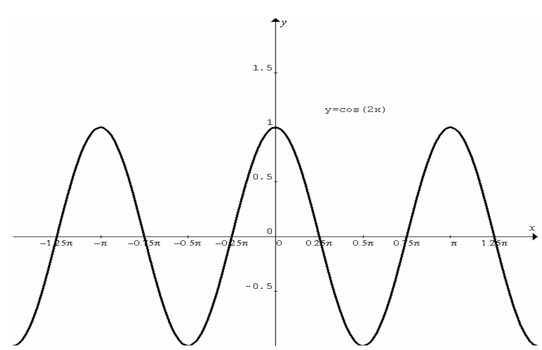



$g\circ f(x)=\sqrt{\sin x}$ is defined when $\sin x\ge 0$ AND $x\in R$,
i.e. $x\in[2n\pi,(2n+1)\pi]$, where $n=0,\pm1,\pm2,......$
Hence $D_{g\circ f}=\{x:2n\pi\le x\le(2n+1)\pi, n=0,\pm1,\pm2,...\}$

Example 2 Find the domain and range, and sketch the graph of $y=|\cos(2x)|$.

$y=|\cos(2x)|$ is a composite function in the form
$y=f\circ g(x)=f(g(x))$, where $f(x)=|x|$ and $g(x)=\cos(2x)$.
For $y=|\cos(2x)|$ to be defined, $\cos(2x)\in R$ AND $2x\in R$,
i.e. $x\in R$. Hence $D_{f\circ g}=R$.
Since $-1\le\cos(2x)\le 1$, $\therefore 0\le|\cos(2x)|\le 1$. Hence the range of the composite function is $R_{f\circ g}=[0,1]$.



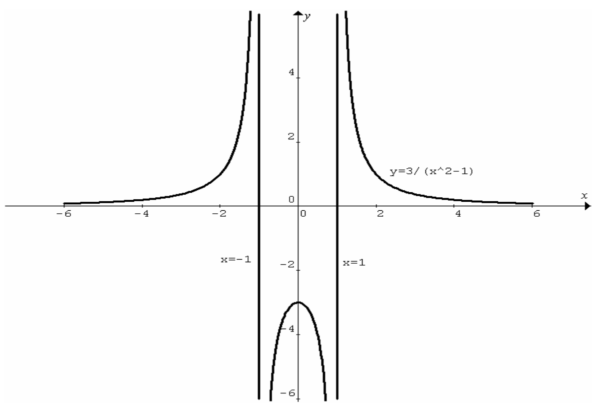




The graph of $y=\cos(2x)$ is also shown for comparison. The negative half is reflected in the x-axis.

Example 3 Find the domain and range, and sketch the graph of $y=\dfrac{3}{x^2-1}$.

This is a composite function of the form $y=f\circ g(x)=f(g(x))$, where $f(x)=\dfrac{3}{x}$ and $g(x)=x^2-1$.

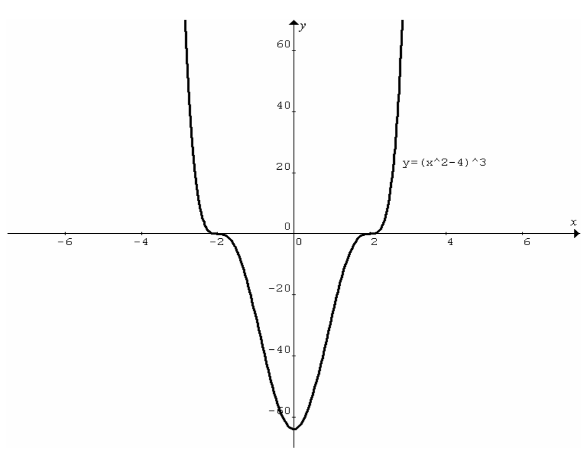
The function is defined when $x^2-1\neq 0$ (i.e. $x\neq \pm 1$) AND $x\in R$. Hence $D_{f\circ g}=R\setminus\{-1,1\}$ and the function has vertical asymptotes $x=-1$ and $x=1$. The value of the function cannot be zero, $\therefore R_{f\circ g}=R\setminus\{0\}$.




Example 4 Find the domain and range, and sketch the graph of $y=(x^2-4)^3$.

This is a composite function of the form $y=f\circ g(x)=f(g(x))$, where $f(x)=x^3$ and $g(x)=x^2-4$. It is defined for all real x values. Hence $D_{f\circ g}=R$. The lowest value of the function is $(-4)^3=-64$. Hence $R_{f\circ g}=[-64,\infty)$.

The function can be expressed as $y=(x^2-4)^3=(x-2)^3(x+2)^3$. $\therefore$ the x-intercepts at $\pm 2$ are stationary points of inflection.




**Graphs of inverse relations**

A relation is a set of points. A new set of points can be generated by interchanging the x and y-coordinates of each point. This new set of points is called the **inverse** of the original relation. The equation of the inverse is obtained by interchanging x and y in the original equation.

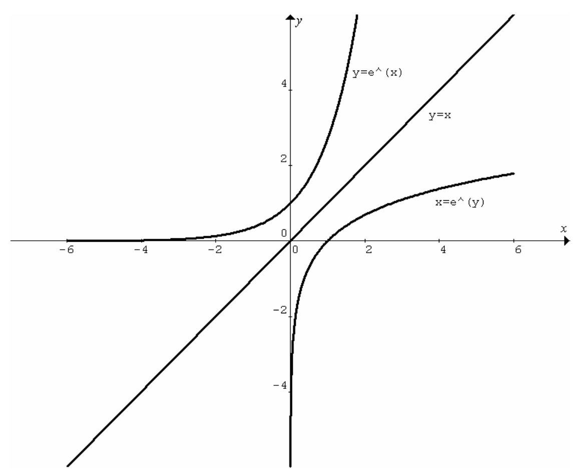
The y-intercept of the original relation becomes the x-intercept of the new relation; the x-intercept of the original becomes the y-intercept of the new.

The horizontal asymptote of the original relation becomes the vertical asymptote of the new relation; the vertical asymptote of the original becomes the horizontal asymptote of the new.

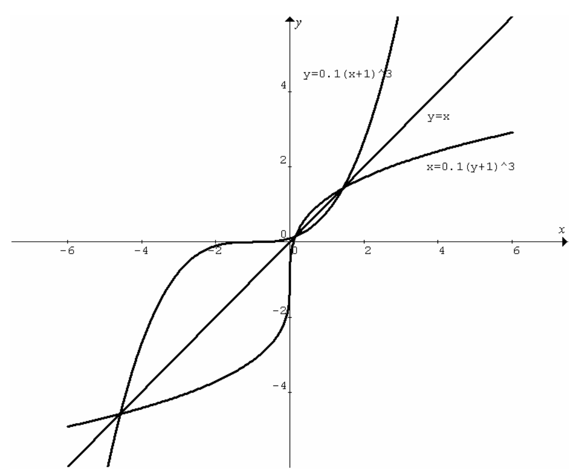
The range of the original relation becomes the domain of the new relation; the domain of the original becomes the range of the new.

Graphically the inverse relation and the original relation are reflections of each other in the line $y=x$. Note that only equal scales for both axes can display the reflection visually.

Example 1




Example 2



**Approximate solutions of equations by graphical method**

Example 1 Use graphics calculator to solve
$2.4x^3 - 7.2x^2 + 5.1 = 0$, correct to four decimal places.

Use graphics calculator to draw $y = 2.4x^3 - 7.2x^2 + 5.1$.
2nd calc zero to find all the x-intercepts: $x = -0.7525$,
$x = 1.0417$, $x = 2.7108$.

Example 2 Solve $y = \dfrac{2}{x^2 + 1}$ and $y = -2x + 1$
simultaneously, correct to four decimal places.

Use graphics calculator to draw the two functions.
2nd calc intersect to find the coordinates of the intersection:
$x = -0.3761$, $y = 1.7522$.

Example 3 Solve $\sin(2x) = x - 1$, correct to 3 decimal places.

It can be solved in two ways using graphical method. Do not forget to set graphics calculator in radian mode.

First way: Draw $y = \sin(2x)$ and $y = x - 1$. 2nd calc intersect to find the x-coordinate(s) of the intersection(s).

Second way: Rewrite the equation to obtain
$\sin(2x) - x + 1 = 0$. Draw $y = \sin(2x) - x + 1$. 2nd calc zero to find the x-intercept(s), $x = 1.377$.

Example 4 Solve $-5\sin(\pi(x - 0.25)) + 3 = 0$, $-1 < x < 3$,
correct to 3 decimal places.

Set graphics calculator in radian mode.
Set the window $x_{\min} = -1$, $x_{\max} = 3$.
Draw $y = -5\sin(\pi(x - 0.25)) + 3$. 2nd calc zero to find the x-intercepts: $x = -0.955$, $x = 0.455$, $x = 1.045$, $x = 2.455$.
Note that the last two solutions can also be obtained by adding 2 to the first two solutions, $\because$ the function has a period of 2.