2004 VCAA Math Methods Exam 1 Solutions

PART I

1	2	3	4	5	6	7	8	9	10
C	C	E	C	B	B	B	D	E	B

11	12	13	14	15	16	17	18	19	20
C	D	B	C	D	C	E	E	A	A

21	22	23	24	25	26	27
D	B	A	D	C	D	A

Q1 $\Pr(odd) = 0.3 + 0.25 = 0.55 \ \therefore$ C

Q2 $E(w) = 0\times 0.2 + 1\times 0.4 + 2\times 0.3 + 3\times 0.1 = 1.3 \ \therefore$ C

Q3 There are 10 purple and 6 green socks.
$\Pr(matching) = 1 - \Pr(notmatching)$

$$= 1 - \frac{{}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2} = \frac{1}{2} \ \therefore \text{E}$$

Q4 $\mu = np = 20$, $\text{var}iance = np(1-p) = 4$,
$\therefore 20(1-p) = 4$, $p = 0.8$, $n = 25 \ \therefore$ C

Q5 $\Pr(X > 15) = \Pr\left(Z > \frac{15 - 12.2}{1.4}\right) = \Pr(Z > 2) \ \therefore$ B

Q6 Since $T = \frac{2\pi}{n}$, $\frac{1}{10} = \frac{2\pi}{n}$, $n = 20\pi \ \therefore$ B

Q7 Period $T = \frac{2\pi}{n} = \frac{2\pi}{\frac{4\pi}{25}} = 12.5$ hours. From low tide to high tide, $\frac{1}{2}T = 6.25$ hours. The time is 6.15 am.
$\therefore$ B

Q8 $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$, $\frac{x}{2} = \frac{\pi}{6}, \frac{11\pi}{6} \ \therefore \ x = \frac{\pi}{3}, \frac{11\pi}{3}$
Sum of the two solutions $= 4\pi \ \therefore$ D

Q9 $ax^3 - bx = x(ax^2 - b) = x\left(\left(\sqrt{a}x\right)^2 - \left(\sqrt{b}\right)^2\right)$
$= x\left(\sqrt{a}x - \sqrt{b}\right)\left(\sqrt{a}x + \sqrt{b}\right) \ \therefore$ E

Q10 g is not a one-to-one function, $\therefore$ B

Q11 $e^{ax} = 0.4$, $ax = \log_e(0.4)$, $x = \frac{\log_e(0.4)}{a} \ \therefore$ C

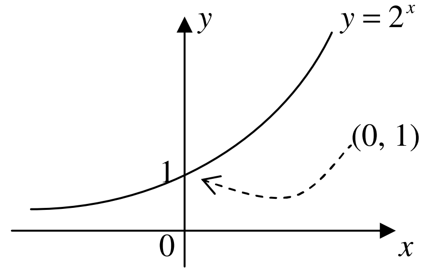
Q12 Graphics calculator, intersection of $y = \log_e(x+1)$ and $y = 1 - x$, $x = 0.5571 \ \therefore$ D

Q13 The quadratic equation $y = P(x-Q)^2 + 1$ is in turning point form where Q is the x-coordinate of the turning point, $Q = 2$. The y-intercept is $\left(0, 2\frac{1}{3}\right)$,

$\therefore 2\frac{1}{3} = P(-2)^2 + 1$, $P = \frac{1}{3} \ \therefore$ B

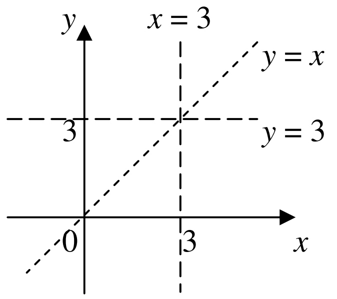
Q14 x-intercepts are at $x = a$ and $x = b$, $\therefore$ $x - a$ and $x - b$ are the two distinct factors of the polynomial function *f*. Since *f* is cubic, it has either $x - a$ or $x - b$ as a repeated factor. $\therefore$ C

Q15



$\therefore$ D

Q16



$\therefore$ C

Q17 $g(x) = 5\cos(3x)$
Dilation from y-axis by factor $\frac{1}{3}$
Dilation from x-axis by factor 5

$\therefore$ E

Q18 $y=(x+1)^3(x-1)+4$ is the vertical translation by +4 of $y=(x+1)^3(x-1)$ which has a stationary point of inflection at $x=-1$.
Hence $y=(x+1)^3(x-1)+4$ has a stationary point of inflection at $x=-1$. $\therefore$ E

Q19 Rate of change of y with respect to x is $\frac{dy}{dx}=2x+2$. At $x=k$, $\frac{dy}{dx}=2k+2$ $\therefore$ A

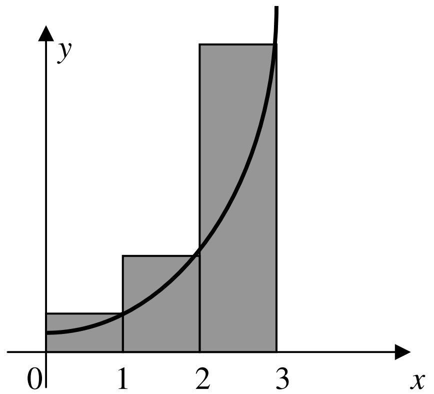
Q20 When $t = 0$, $N = 1000$, when $t = 10$, $N = 1000e$.
Average rate $=\frac{1000e-1000}{10-0}=171.83$ $\therefore$ A

Q21 $h=1.8-2=-0.2$, $x=2$.
$\therefore$ $1.8^3=f(1.8)\approx f(2)-0.2f'(2)$ $\therefore$ D

Q22 $f'(x)$ is always positive, $f'(x)$ increases as x increases, and $f'(x)\neq 0$ at $x=0$. $\therefore$ B

Q23 Since $f(x)$ is cubic, $\therefore \int f(x)dx$ is quartic.
For $x>0$, the gradient of $y=\int f(x)dx$ i.e. $f(x)$ is positive. $\therefore$ A

Q24



x	1	2	3
y	2	9	28

Shaded area $1\times 2+1\times 9+1\times 28=39$ $\therefore$ D

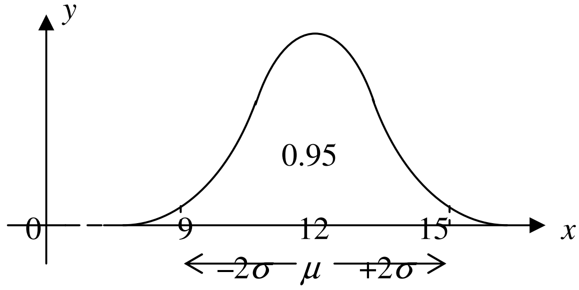
Q25 Between $x=a$ and $x=b$ the region bounded by the curve and the x-axis is below the x-axis. For this region the area is $-\int_a^b f(x)dx$. $\therefore$ C

Q26 When $y=2$, $e^x=2$, $\therefore x=\log_e 2$. $\therefore$ D

Q27 $\int_0^{1.5}\frac{1}{2x+1}dx=\left[\frac{\log_e(2x+1)}{2}\right]_0^{1.5}=\frac{1}{2}\log_e 4$
$=\log_e 4^{\frac{1}{2}}=\log_e 2$. $\therefore$ A

PART II

Q1a



y: normal distribution $N(\mu=12,\sigma=1.5)$
x: random variable, diameter (cm)
μ: mean
σ: standard deviation

Q1b $\Pr(x<k)=0.1600$,
k = inversenorm(0.16,12,1.5) = 10.5

Q2a $g(x)=f(x-3)=(x-3)^2$

Q2b $h(x)=g(x)-1=(x-3)^2-1$

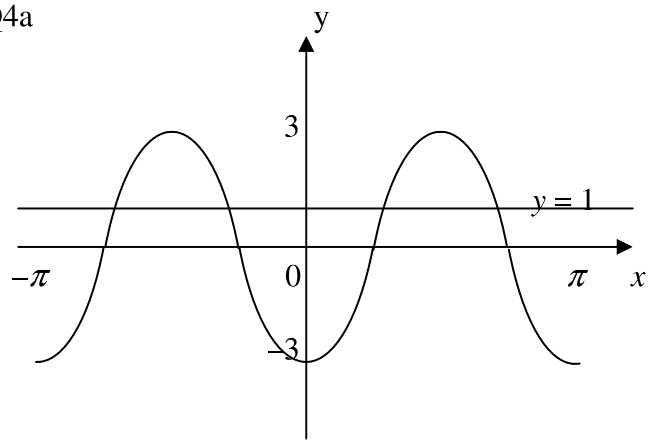
Q2c $k(x)=h(2x)=(2x-3)^2-1$

Q3a Domain $(1,\infty)$, range R.

Q3b f^{-1} exists because f is one-to-one.

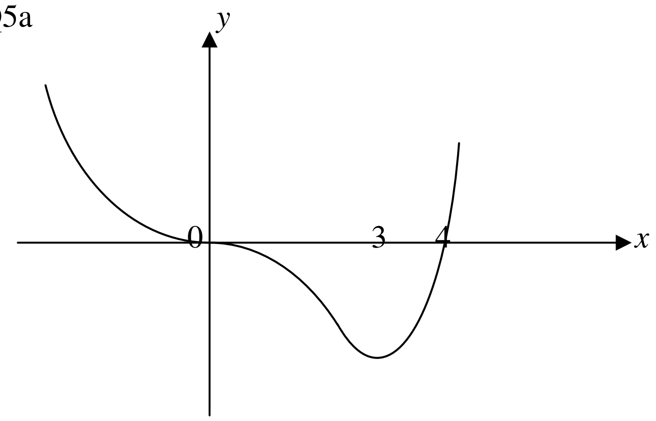
Q3c Let $y=0.5\log_e(x-1)$, then the equation of f^{-1} is $x=0.5\log_e(y-1)$. Express y in terms of x.
$\log_e(y-1)=2x$, $y-1=e^{2x}$, $y=e^{2x}+1$.
$\therefore f^{-1}(x)=e^{2x}+1$.

Q4a



Q4b Draw horizontal line $y=1$, there are four intersections $\therefore$ 4 solutions.

Q5a



Q5b x-intercept at $x=4$, $\therefore b=4$ and $f(x)=ax^3(x-4)$. The curve passes through $(2,-4)$, $\therefore f(2)=a(2)^3(2-4)=-4$, $\therefore a=\frac{1}{4}$.

Q5c $f(x)=\frac{1}{4}x^3(x-4)=\frac{1}{4}x^4-x^3$,
$f'(x)=x^3-3x^2$. At $x=4$,
gradient of the tangent $m=f'(4)=64-48=16$.
$\therefore$ equation of the tangent is $y-0=16(x-4)$,
i.e. $y=16x-64$.

Q6a Binomial: $n=4$, p is probability of success (i.e. selecting a rainbow-coloured ball). Let X (number of successes) be the random variable,
$\Pr(X=1)={}^4C_1 p(1-p)^3=4p(1-p)^3$

Q6b $\frac{d\Pr(X=1)}{dp}=-4p\times 3(1-p)^2+4(1-p)^3$
$=(1-p)^2(-12p+4(1-p))$
$=(1-p)^2(4-16p)$

Max. probability when $\frac{d\Pr(X=1)}{dp}=0$,
i.e. $(1-p)^2(4-16p)=0$. Since $p\neq 1$, $\therefore 4-16p=0$,
$\therefore p=\frac{1}{4}$.

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