# 2016 Specialist Mathematics Problem Solving Task

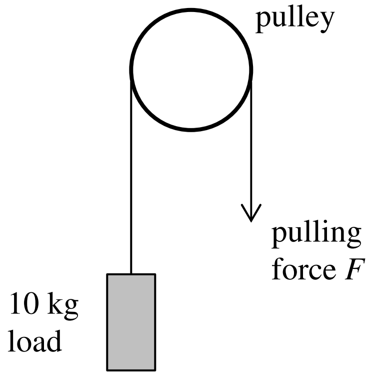
**You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator, 5 min reading + 120 min writing**
***Working must be shown for questions worth 2 or more marks.***

**Theme: Force required moving a load at constant speed**

Distance is in metres, mass in kilogram, time in seconds and force in newtons.
$g = 9.8 \text{ N kg}^{-1}$

**Problem 1**

A 10 kg load is raised by pulling a rope over a frictionless pulley.




**a.** Determine the pulling force required to raise the load at a constant speed of $0.2 \text{ m s}^{-1}$. 1 mark

**b.** What is the tension in the rope while the load is raised at $0.2 \text{ m s}^{-1}$? 1 mark

**c.** Determine the pulling force required to lower the load at a constant speed of $0.2 \text{ m s}^{-1}$. 1 mark

**d.** What is the tension in the rope while the load is lowered at $0.2 \text{ m s}^{-1}$? 1 mark

**e.** If the load is $M$ kg, write down a formula for pulling force $F$ required to raise the load at constant speed.

1 mark

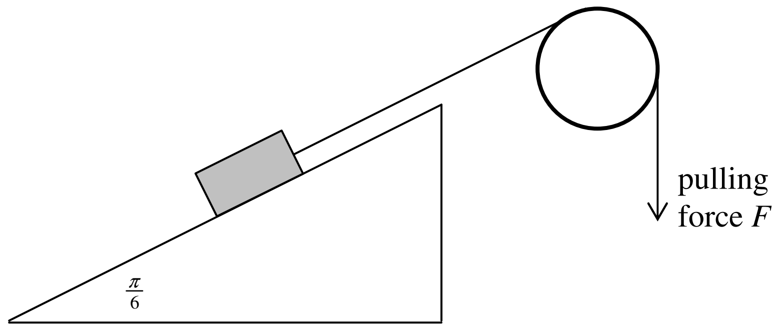
**f.** If the load is $M$ kg, write down a formula for pulling force $F$ required to lower the load at constant speed.

1 mark

**Problem 2**

Now the 10 kg load is placed on a rough inclined plane. The inclined plane makes an angle $\frac{\pi}{6}$ with a horizontal plane.

The force of friction between the load and the inclined plane is given by $F_{\text{friction}} = 0.25N$ where $N$ is the normal reaction force of the inclined plane on the load.




**a.** Calculate the value of $F_{\text{friction}}$. 3 marks

**b.** Determine the pulling force required to raise the load at a constant speed of $0.2 \text{ m s}^{-1}$. 2 marks

**c.** Determine the pulling force required to lower the load at a constant speed of $0.2 \text{ m s}^{-1}$. 2 marks

Now the inclined plane makes an angle of $\theta$ with the horizontal plane, and $0 \le \theta \le \frac{\pi}{2}$.

The force of friction between the load and the inclined plane is given by $F_{\text{friction}} = \mu N$ where $\mu > 0$.

**d.** If the load is $M$ kg, show that a formula for pulling force $F$ required to raise the load at constant speed is $F = Mg(\mu\cos\theta + \sin\theta)$.

3 marks

**e.** If the load is $M$ kg, in terms of $M$, $g$, $\theta$ and $\mu$, write down a formula for pulling force $F$ required to lower the load at constant speed.

2 marks

**f.** By CAS express $\theta$ in terms of $M$, $g$, $F$ and $\mu$, where $F$ is the pulling force required to raise the load at constant speed.

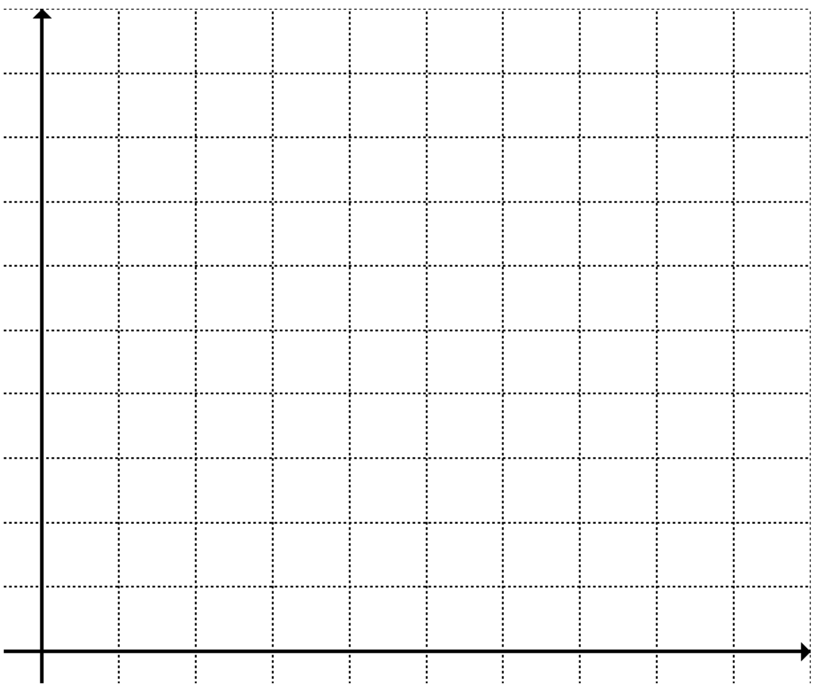
2 marks

**g.** If $M = 10\,\text{kg}$, $\mu = 0.25$ and $\theta < \frac{\pi}{2}$, what is the value of $\theta$ which requires a pulling force of 98 newtons to raise the load at constant speed?

2 marks

**h.** If $\mu = 0.25$, sketch the graph of $\left(\frac{F}{Mg}\right)$ versus $\theta$, where $F$ is the required force to raise the load at constant speed. Show the coordinates of the endpoints.

3 marks



**i.** Describe and explain the situation represented by each end point. 2 marks

Use your graph to help to find the answers to parts **j**, **k** and **l**.

**j.** If $M = 10$ kg, what is the maximum force required to raise the load at constant speed? 1 mark

**k.** At what angle the maximum force is required to raise the load at constant speed? 1 mark

**l.** If $M = 15$ kg, at what angle the maximum force is required to raise the load at constant speed? 1 mark

**m.** Differentiate $F = Mg(\mu\cos\theta + \sin\theta)$ with respect to $\theta$. Do not use CAS. 2 marks

**n.** Hence show the angle $\theta$ which requires maximum force to raise the load at constant speed is given by $\theta = \tan^{-1}\left(\frac{1}{\mu}\right)$. 2 marks

**o.** Hence verify your answer to part **k**. 1 mark

**p.** Show that the maximum force required to raise the load at constant speed is given by $F = Mg\sqrt{1+\mu^2}$ .

4 marks

**q.** Hence verify your answer to part **j**. 1 mark

**r.** The rope used to raise the load will break if its tension exceeds $10\sqrt{2}\,g$ newtons. Given $\mu = 1$ and the inclined plane makes an angle $\frac{\pi}{6}$ with the horizontal, determine the maximum mass of the load that can be raised by the rope.

2 marks

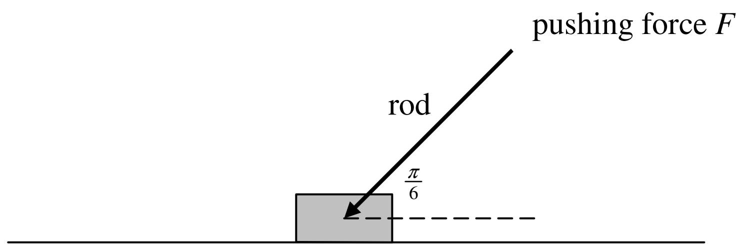
**s.** The rope used to raise the load will break if its tension exceeds $10\sqrt{2}\,g$ newtons. The inclined plane makes an angle $\theta$ with the horizontal, which requires *maximum* pulling force $F$ to raise the load. Given $\mu = 1$, determine the maximum mass of the load that can be raised by the rope.

2 marks

**Problem 3**

Consider the 10 kg load on a horizontal plane. A rod (of negligible mass) is attached to the load, making an angle $\frac{\pi}{6}$ with the horizontal plane. A pushing force $F$ is applied to the rod.

The force of friction between the load and the horizontal plane is given by $F_{\text{friction}} = 0.25N$ where $N$ is the normal reaction force of the plane on the load.




**a.** Use the vertical force components to show that $N = \frac{F}{2} + 98$. 2 marks

**b.** Find the force of friction $F_{\text{friction}}$ between the load and the horizontal plane in terms of pushing force $F$.

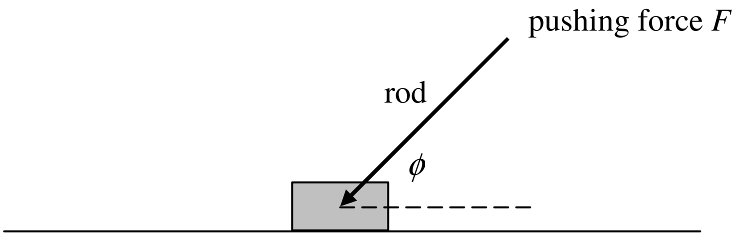
1 mark

**c.** What is the pushing force required to move the load at a constant speed of $0.2\ \text{m s}^{-1}$ along the plane?

2 marks

Now the pushing force $F$ makes an angle of $\phi$ with the horizontal plane, where $0 \le \phi \le \frac{\pi}{2}$.

The force of friction between the load and the plane is given by $F_{\text{friction}} = \mu N$, and $\mu > 0$.




**d.** If the load is $M$ kg, show that a formula for pushing force $F$ required to move the load at constant speed along the plane is $F = \dfrac{\mu M g}{\cos\phi - \mu \sin\phi}$. 3 marks

**e.** Use the above formula to verify part **c**. 1 mark

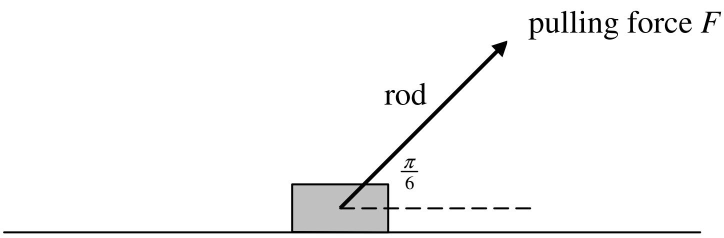
**f.** In terms of $\mu$ **only**, find the values of $\phi$ at which it is impossible to move **any** load at constant speed along the plane.

5 marks

**Problem 4**

Consider a 10 kg load on a horizontal plane. A rod (of negligible mass) is attached to the load, making an angle $\frac{\pi}{6}$ with the horizontal plane. This time a pulling force is applied to the rod.

The force of friction between the load and the horizontal plane is given by $F_{\text{friction}} = 0.25N$ where $N$ is the normal reaction force of the plane on the load.




**a.** Use the vertical force components to show that $F = 196 - 2N$. 1 mark

**b.** Find the maximum pulling force $F$ which will not lift the load off the plane. 1 mark

**c.** Find the force of friction $F_{\text{friction}}$ between the load and the horizontal plane in terms of pulling force $F$.

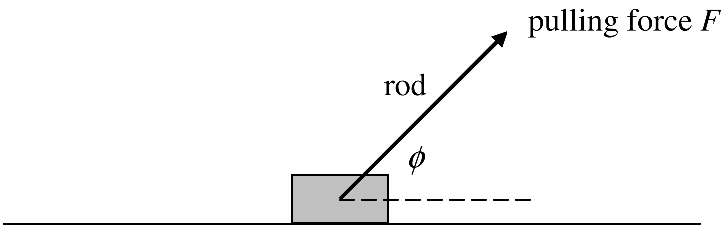
1 mark

**d.** What is the pulling force required to move the load at a constant speed of $0.2 \text{ m s}^{-1}$ along the plane?

2 marks

Now the pulling force $F$ makes an angle of $\phi$ with the horizontal plane, where $0 \le \phi \le \frac{\pi}{2}$.

The force of friction between the load and the plane is given by $F_{\text{friction}} = \mu N$, and $\mu > 0$.
Assume that the load is never lifted off the plane.




**e.** If the load is $M$ kg, show that a formula for pulling force $F$ required to move the load at constant speed along the plane is $F = \dfrac{\mu Mg}{\cos\phi + \mu\sin\phi}$. 2 marks

**f.** Use the above formula to verify part **d**. 1 mark

**g.** Show that $\cos\phi + \mu\sin\phi \ge \mu$ for the load to move at constant speed along the plane. 3 marks

**h.** For $\mu > 1$, find the interval for $\phi$ that allows the load to move at constant speed along the plane. Show working with clear explanation.

2 marks

**i.** Interpret and explain the situations represented by the endpoints (inclusive and/or exclusive) of the interval for $\phi$ in part **h**.

3 marks

**j.** For $\mu \le 1$, find the interval for $\phi$ that allows the load to move at constant speed along the plane. Show working with clear explanation.

3 marks

**k.** Interpret and explain the situations represented by the endpoints (inclusive and/or exclusive) of the interval for $\phi$ in part **j**.

3 marks

## End of Task