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2016
Mathematical
Methods

Year 12
Modelling Task

Time allowed: 2 hours

You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator

Working must be shown for questions worth 2 or more marks. Total: 70 marks

Theme: Random sampling

Question 1 A small bag contains 5 red and 3 green jelly beans.

a. In how many ways can you select 2 red and 2 green jelly beans **without** replacement? 2 marks

b. If the selection of 4 jelly beans is done at random (sampling **without** replacement), what is the exact probability of obtaining a sample containing 2 red and 2 green jelly beans, considering the possible outcomes are equally likely? 1 mark

c. Draw a tree diagram to illustrate the selection process of 4 jelly beans one at a time **without** replacement, and indicate the conditional probability next to each branch. Use it to find the exact probability of obtaining a sample containing 2 red and 2 green jelly beans. 6 marks

d. If the selection of the 4 jelly beans one at a time **without** replacement is done at random, what is the exact probability of obtaining a sample containing 2 or more red jelly beans? Use the tree diagram in part c to answer the question.

3 marks

Question 2 A large bag contains 250 red and 150 green jelly beans.

a. In how many ways can you select 2 red and 2 green jelly beans **without** replacement?

1 mark

b. If the selection of 4 jelly beans is done at random **without** replacement, what is the exact probability of obtaining a sample containing 2 red and 2 green jelly beans, considering the possible outcomes are equally likely?

1 mark

c. If the selection of 4 jelly beans one at a time is done at random **without** replacement, what is the exact probability of obtaining a sample containing 2 or more red jelly beans?

2 marks

d. In how many ways can you select, one at a time **with** replacement, 2 red and 2 green jelly beans, i.e. take a jelly bean out and put it back in the bag after its colour is noted?

1 mark

e. If the selection of 4 jelly beans one at a time is done at random (sampling **with** replacement), what is the exact probability of obtaining a sample containing 2 red and 2 green jelly beans, considering the possible outcomes are equally likely?

1 mark

f. If the selection of 4 jelly beans is done at random **with** replacement, what is the probability of obtaining a sample containing 2 or more red jelly beans?

1 mark

g. Compare and comment on your answers to part c and part f.

1 mark

On a particular school day at a secondary school there are 10 students in Class 12 A (25 students in the class) not wearing proper school uniform.

Question 3

A random sample of 5 students is taken from Class 12 A.

Let X be the random variable: number of students in a sample not wearing proper school uniform, and \hat{P} be the random variable: proportion of students in the sample not wearing proper school uniform.

a. Complete the following table to show the sampling distribution of X and \hat{P} . 6 marks

X						
\hat{P}						
$\Pr(X = x) = \Pr(\hat{P} = \hat{p})$						

b. Use the **definition of expectation of a random variable** to calculate the proportion of students in the sample not wearing proper school uniform that you are expected to find.

2 marks

c. Use the **definition of expectation of a random variable** to calculate the expected value of \hat{P}^2 . Hence find the standard deviation of \hat{P} about the mean of \hat{P} .

3 marks

Question 4 In this question consider Class 12 A as a random sample of 25 students from the whole secondary school.

a. Calculate the 95% confidence interval for p , the proportion of students in the whole school not wearing proper school uniform on the particular school day.

2 marks

b. Find the interval for X that you are 95% certain contains the true number of students not wearing proper school uniform on the particular school day.

1 mark

In fact the whole school has 600 students and 240 of them not wearing proper school uniform.

A random sample of 5 students is taken from the school.

Let X be the random variable: number of students in a sample not wearing proper school uniform, and \hat{P} be the random variable: proportion of students in the sample not wearing proper school uniform.

Question 5

a. Complete the following table, using the binomial distribution to model the sampling distribution of \hat{P} .

5 marks

\hat{P}						
$\Pr(X = x) = \Pr(\hat{P} = \hat{p})$						

b. Use formulas to determine the values of $E(\hat{p})$ and $sd(\hat{p})$.

2 marks

c. Use the binomial distribution as the model to find the probability that $\hat{p} > 0.3$ in a sample.

2 marks

d. Out of 20 random samples of 5 students from the school, how many of them (correct to the nearest whole number) have $\hat{p} \in [0, 0.3)$? Use the binomial model to calculate the answer.

3 marks

e. Use the normal distribution to find (i) the probability that $\hat{p} > 0.3$ in a sample, and (ii) out of 20 random samples of 5 students from the school, how many of them (correct to the nearest whole number) have $\hat{p} \in [0, 0.3)$.

2 + 2 = 4 marks

f. Compare and comment on your answers to part c, part d and part e.

1 mark

Now a random sample of 25 students is taken from the school.

Let X be the random variable: number of students in a sample not wearing proper school uniform, and \hat{P} be the random variable: proportion of students in the sample not wearing proper school uniform.

g. Use the binomial distribution to find (i) the probability that $\hat{p} > 0.3$ in a sample, and (ii) out of 20 random samples of 25 students from the school, how many of them (correct to the nearest whole number) have $\hat{p} \in [0, 0.3)$.

2 + 2 = 4 marks

h. Use the normal distribution to find (i) the probability that $\hat{p} > 0.3$ in a sample, and (ii) out of 20 random samples of 25 students from the school, how many of them (correct to the nearest whole number) have $\hat{p} \in [0, 0.3)$.

2 + 2 = 4 marks

i. Compare and comment on your answers to part g and part h.

1 mark

Another large secondary school (over 1000 students) also has the problem of students not wearing proper school uniform.

The school principal wants to find out the proportion of the student population at the school not wearing proper school uniform.

A random sample of 100 students is taken and there are 35 students not wearing proper school uniform.

Question 6

a. Give an estimate of the proportion of the student population at the school not wearing proper school uniform.

1 mark

b. Give a 95% confidence interval estimate of the proportion of the student population at the school not wearing proper school uniform.

2 marks

c. Give a 80% confidence interval estimate of the proportion of the student population at the school not wearing proper school uniform.

3 marks

d. Compare and comment on your answers to part b and part c in terms of confidence level.

1 mark

A random sample of 200 students is taken and there are 70 students not wearing proper school uniform.

e. Give an estimate of the proportion of the student population at the school not wearing proper school uniform.

1 mark

f. Give a 80% confidence interval estimate of the proportion of the student population at the school not wearing proper school uniform.

1 mark

g. Compare and comment on your answers to part c and part f in terms of sample size.

1 mark

End of task