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Further Mathematics

2014

Trial Examination 2

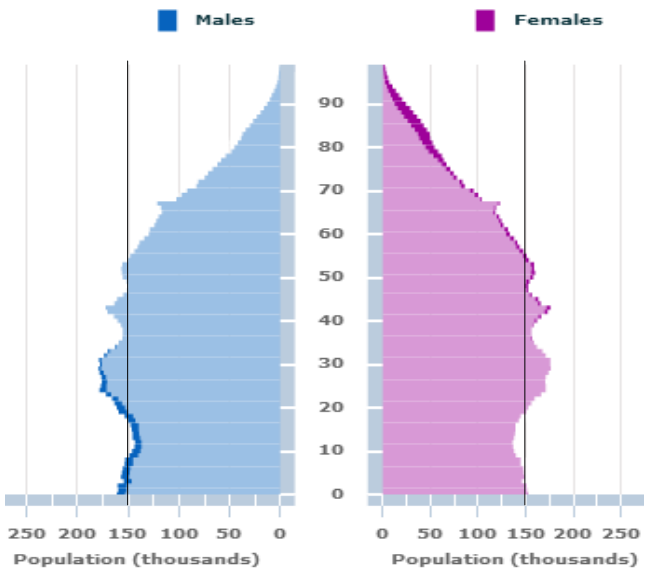
Core – Data analysis
Module 2 – Geometry and trigonometry
Module 3 – Graphs and relations
Module 4 – Business-related mathematics

Instructions:
Answer all questions in the core and the three modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core – Data analysis

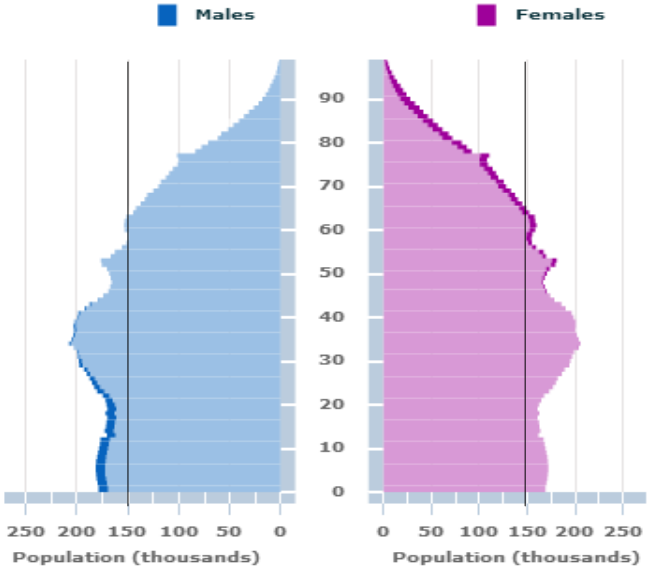
Question 1

The following graphs show the male and female Australian populations in 2014 and the projected male and female populations in 2024. Age groups are shown in each stem. Darker shade indicates excess.



Year 2014

Source: abs



Year 2024

a. In 2014 which age group has the same number of males and females? 1 mark

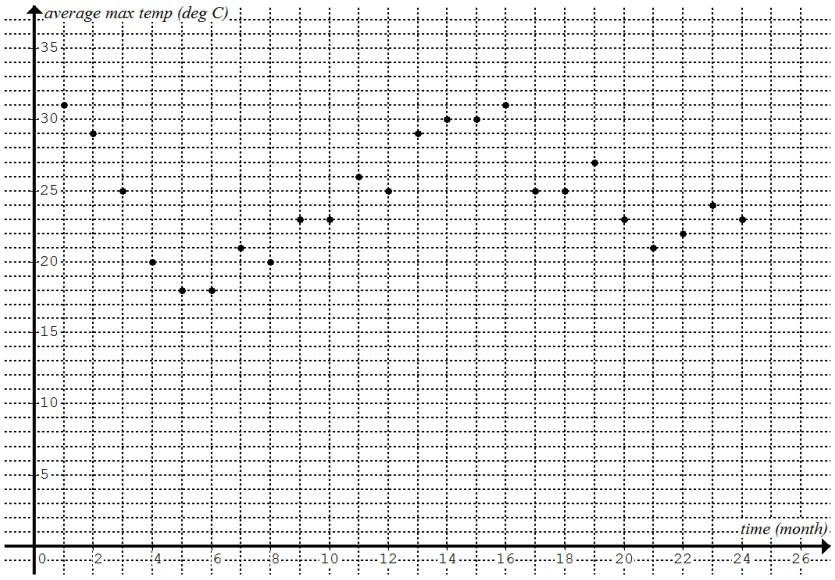
b. What is the projected increase in the number of 70 year old in 2024 in comparison with the number of 70 year old in 2014? Correct your answer to the nearest 10 thousands. 1 mark

c. Australian males have a shorter life span than females. Use certain features/data of the graphs to support the statement. Ignore the effects of immigration in your answer.

2 marks

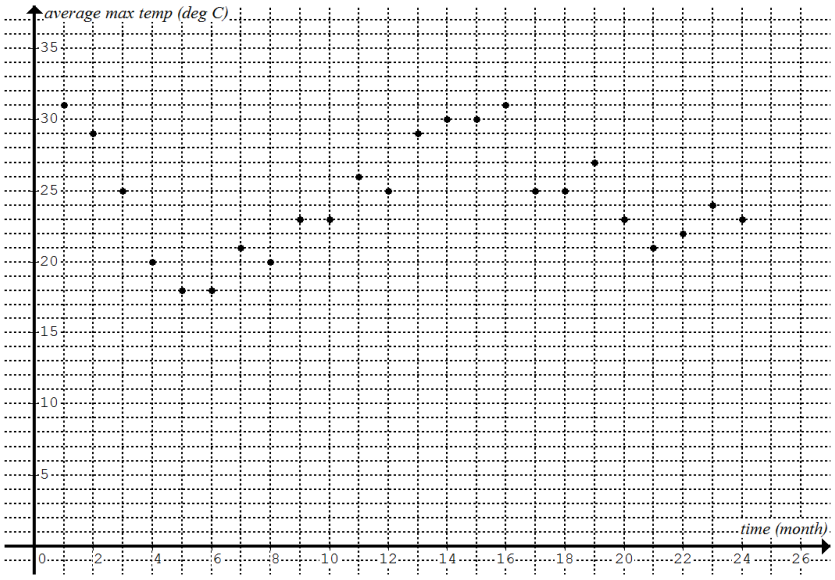
Question 2

The following graph shows the average daily maximum temperatures of a Victorian town over each month in 2012 and 2013. 1 stands for January 2012, 2 for February 2012, ……, 15 for March 2013 etc.



a. Perform a 5-point median smoothing on the data points graphically. 2 marks

b. Draw the 3-median trend line based on the 2012 and 2013 data. 1 mark



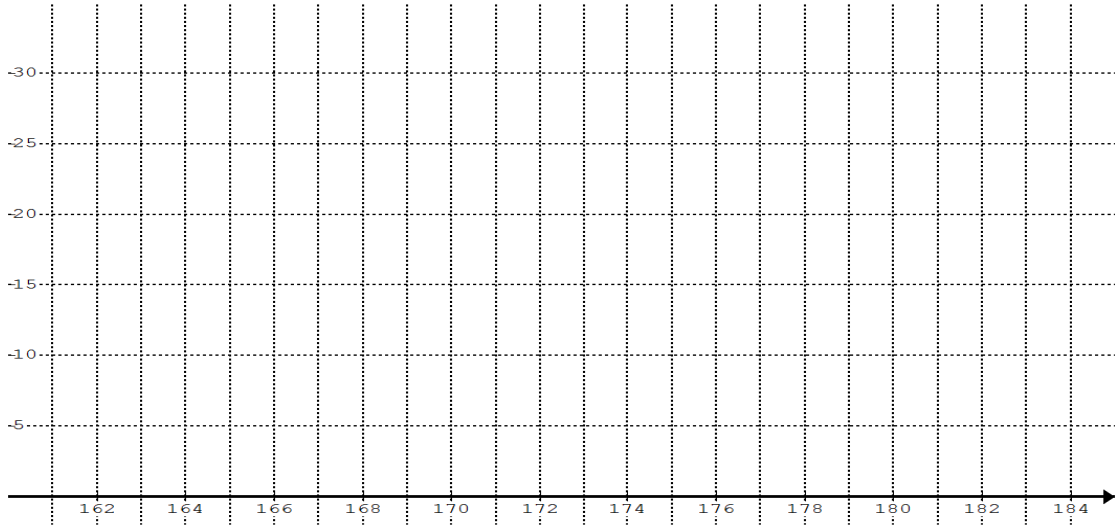
c. Find the equation of the 3-median trend line based on the 2012 and 2013 data. 1 mark

Question 3

The heights of 100 students are presented in the following table.

Height (± 0.5 cm)	Number of students
170	1
171	1
172	5
173	10
174	20
175	27
176	20
177	11
178	3
179	2

a. Construct a suitable histogram of the data in the table. 1 mark



The bell shaped data can be modeled by the normal distribution.

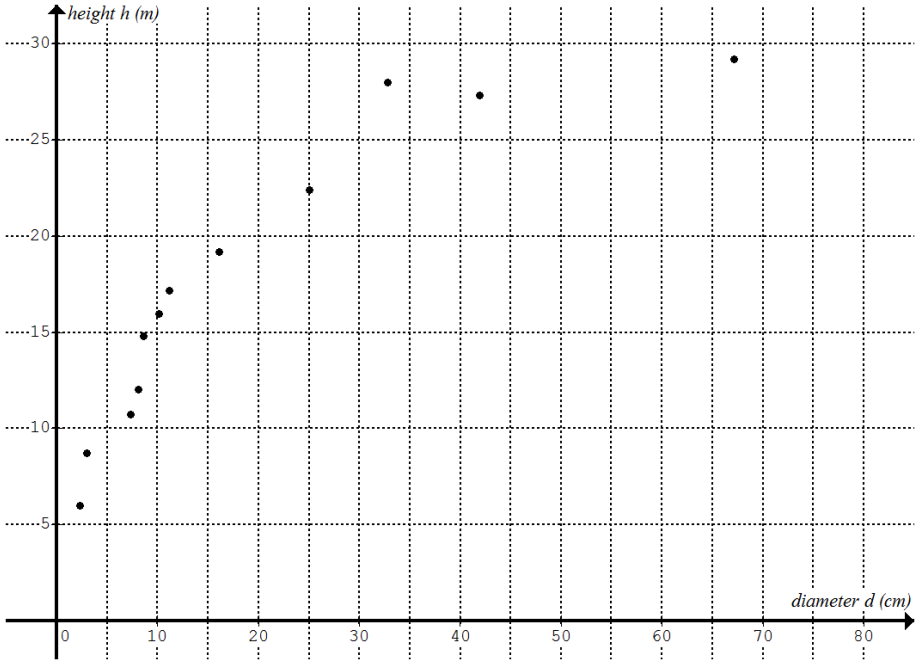
b. Estimate the standard deviation of the data. 1 mark

c. Estimate the z score of 173.5 cm. 1 mark

Question 4

A student measures the diameters (1.5 m above the ground) and the heights of 12 trees of the same species. The results are shown in the table and the scatterplot.

Diameter d (cm)	2.3	3.0	7.3	8.1	8.6	10.1	11.2	16.1	25.0	32.8	41.9	67.1
Height h (m)	6.0	8.7	10.7	12.0	14.8	16.0	17.2	19.2	22.4	28.0	27.3	29.2



a. State an appropriate linearising transformation. 1 mark

b. Determine a straight-line regression to the transformed data. 2 marks

c. What % of variability can be explained by the regression line? 1 mark

Module 2: Geometry and trigonometry

Question 1 The following image shows three kite surfers A, B and C.
A and B are 25 m apart. $\angle BAC = 40°$ and $\angle ABC = 80°$.

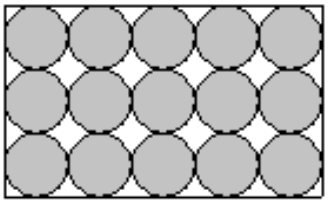
(Source: Flickr)

a. How far is C from A? 2 marks

b. Calculate the area of the region enclosed by ΔABC. 1 mark

Question 2

A rectangular container can fit in n layers of identical solid spheres.
Each layer contains l rows and m columns of spheres. The spheres forming the perimeter of each layer touch the wall(s) of the container. All spheres in the top layer touch the lid.
The radius of each sphere is r units.
The following diagram shows the arrangement of a layer of spheres consisting of 3 rows and 5 columns.



a. Calculate the exact value of the ratio $\dfrac{\text{total volume of the } l \times m \times n \text{ spheres}}{\text{total volume of the remaining empty space inside the container}}$.

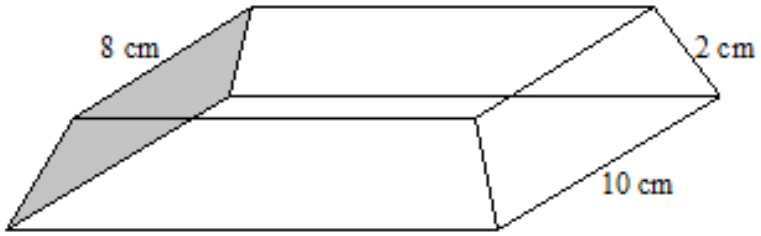
2 marks

b i. A second container can just fit in an extra layer of spheres.
Determine the exact value of the ratio $\dfrac{\text{total volume of the } l \times m \times (n+1) \text{ spheres}}{\text{total volume of the remaing empty space inside the container}}$. 1 mark

b ii. Given $n = 10$, calculate the exact value of the ratio $\dfrac{\text{volume of the second container}}{\text{volume of the first container}}$. 1 mark

Question 3

A frustum of a regular square base pyramid is shown below. It is placed on a horizontal surface so that the top and the four sloping faces are exposed to the air.



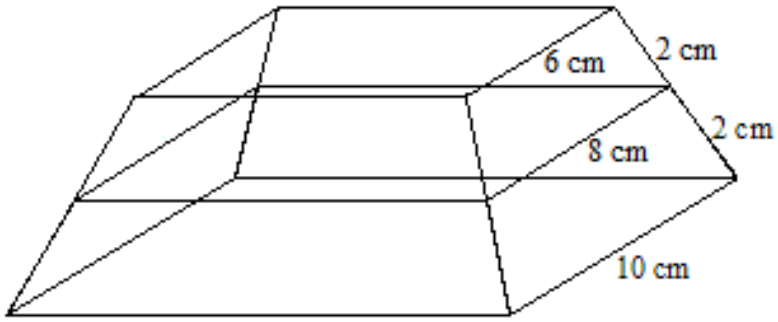
a. Calculate the angle (in degree, 1 decimal place) between the shaded sloping face and the square base.

2 marks

b Calculate the exact height of the pyramid from which the frustum is formed. 2 marks

c. Calculate the exact volume (cm^3) of the frustum. 2 marks

A second frustum is placed on top of the first to form a larger frustum as shown in the following diagram.

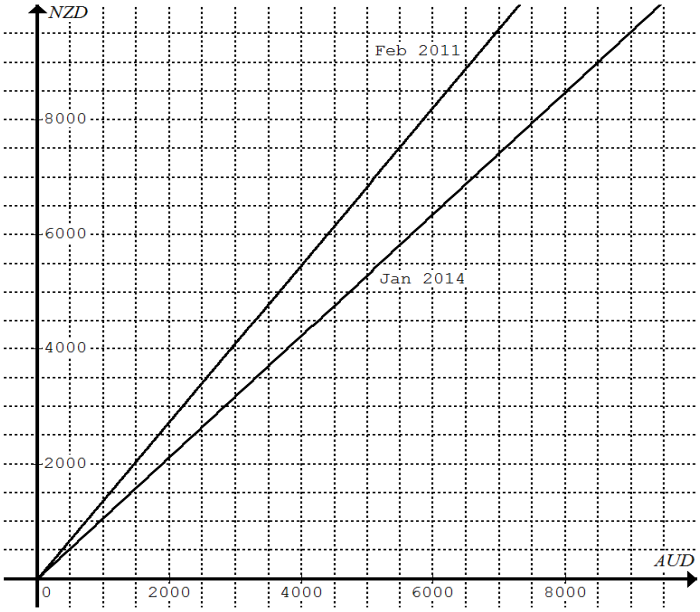


d. Determine the exact difference in area of the exposed surfaces of the larger frustum and the original smaller frustum. Note: The base is not exposed. 2 marks

Module 3: Graphs and relations

Question 1

Reserve Bank of Australia official exchange rate of Australian dollar (AUD) to New Zealand dollar (NZD) varies with time. The rate was highest in Feb 2011 and lowest in Jan 2014 in the past five years. The following graphs show the highest rate in Feb 2011 and the lowest rate in Jan 2014.



a. 9000 NZD could buy x AUD. Find the highest value of x in Jan 2014 according to Reserve Bank of Australia official exchange rate. 1 mark

b. How much more (largest amount) AUD could be bought with 9000 NZD in Jan 2014 than in Feb 2011? Correct your answer to the nearest hundred dollars. 1 mark

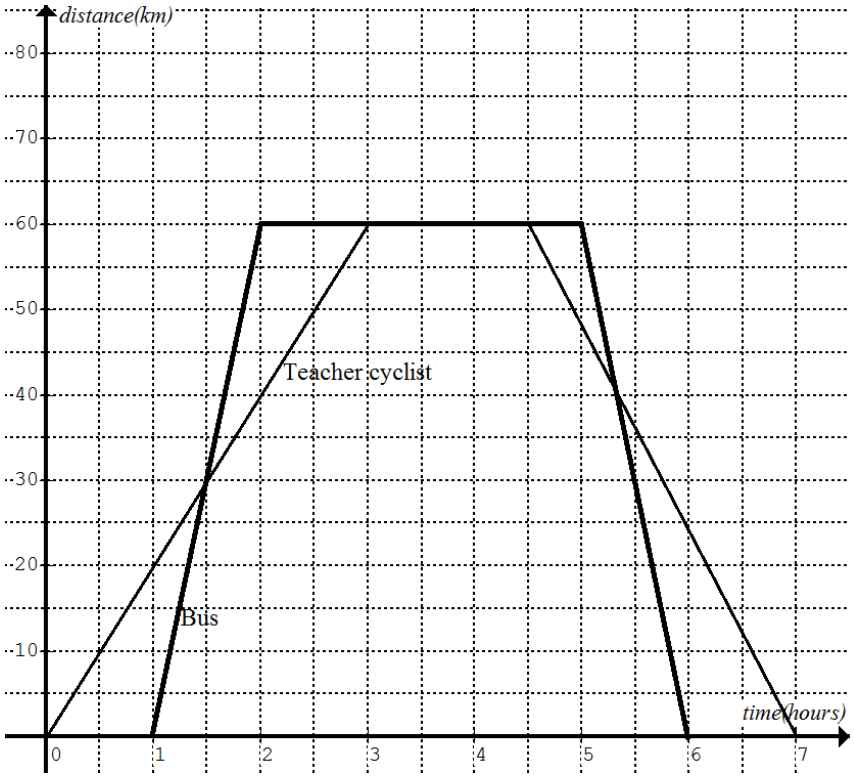
c. Write an equation for changing AUD to NZD at the lowest rate in Jan 2014. 1 mark

d. A money exchanger charges 10% fee on the money that you wish to change to a foreign currency. How much (smallest amount) NZD could you get for 8500 AUD in Jan 2014? 1 mark

e. Draw a line on the above grid which could be used to determine the smallest amount in NZD for an amount in AUD in Jan 2014 by the money exchanger, with the 10% fee included. 1 mark

Question 2

A school bus carries students on an excursion. The following distance-time graph shows the distance of the bus from the school. A teacher starts cycling to the same destination an hour earlier and returns to the school an hour later than the bus.



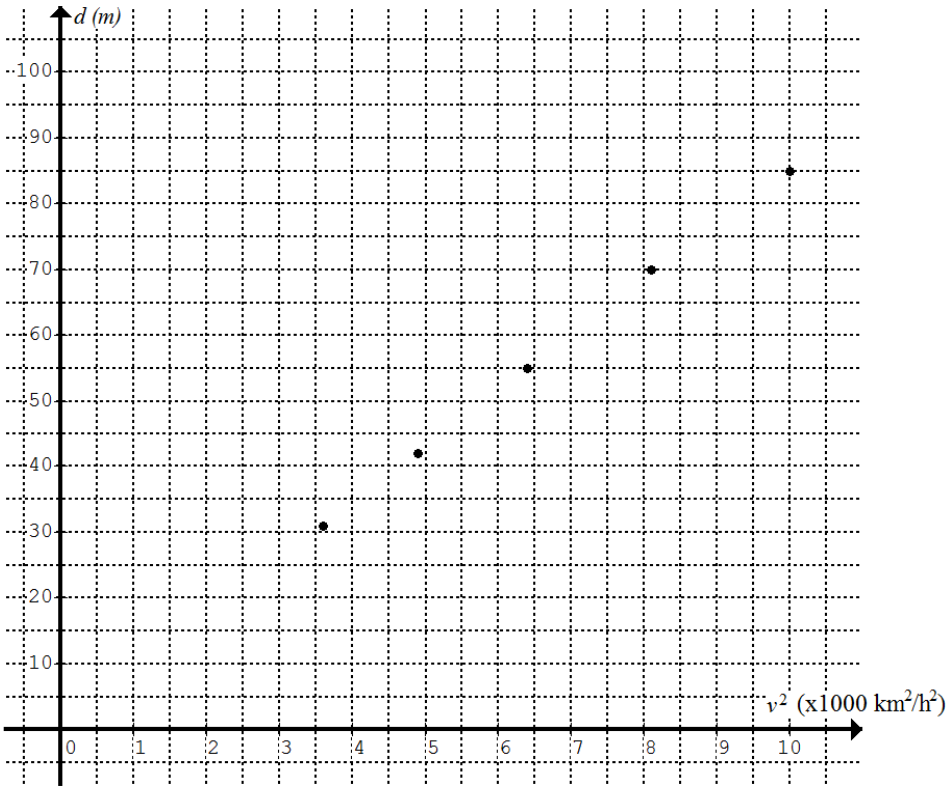
a. How far is the destination from the school? 1 mark

b. For how long does the teacher (the cyclist) stay with the students at the destination? 1 mark

c. Calculate the average speed of the bus from the time it leaves the school to the time it returns to the school, including the time when the bus is parked at the destination. 1 mark

Question 3

The following d vs v^2 graph shows the relationship between braking distance d (metres) and speed v (kilometres per hour) of a car.



a. Find the braking distance when the car speed is 50 km/h. 1 mark

b. Find the speed of the car if the braking distance is 60 m. 1 mark

c. Find the equation relating d and v. 1 mark

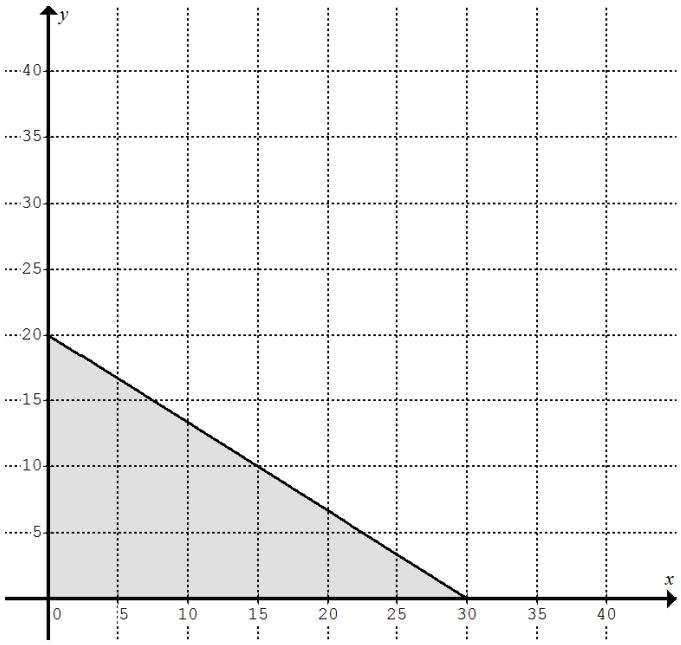
Question 4

Two alloys A and B are manufactured with different mixtures of two metals M_1 and M_2.
The proportion of M_1 and M_2 in alloy A is 0.5 and 0.5.
The proportion of M_1 and M_2 in alloy B is 0.25 and 0.75 respectively.
The daily supply of M_1 is 10 tonnes, and the daily supply of M_2 is 15 tonnes.
The net profit is \$600 per tonne from alloy A, and it is \$500 per tonne from alloy B.

Let x tonnes be the daily production of alloy A, and y tonnes the daily production of alloy B.

a. Three of the constraints on x and y are represented by $x \geq 0$, $y \geq 0$ and $0.5x + 0.75y \leq 15$.
Write down one more constraint on x and y. 1 mark

b. The shaded region in the following graph is defined by the three given constraints in part a.
Sketch the fourth constraint on x and y in the same graph. 1 mark



c. State the profit function \$$P$ when x tonnes of alloy A and y tonnes of alloy B are manufactured and sold.
1 mark

c. Determine the maximum daily profit achievable by the manufacturer. 1 mark

Module 4: Business-related mathematics

Question 1

Two companies provide mobile broadband service. Optra charges $130 prepaid for 15GB of data 1 year expiry, and Telstus charges $200 for 12 GB.

a. Calculate the average cost per month if you use Optra service and no recharge is required. 1 mark

b. By how much per GB of data does Telstus charge more than Optra? 1 mark

Question 2

A smart 15 year old student starts saving $100 at the beginning of each month for the next 15 years, receiving 8% p.a. of interest compounded monthly.

a. Calculate the total in the account after interest is added at the end of the third month. 1 mark

b. Calculate the total in the account after interest is added at the end of the 15 years. 1 mark

c. Calculate the total interest amount over the 15 year period. 1 mark

Question 3

The cost of an electric toaster was $6 to a shopkeeper. It was marked up to 300% of the cost to cover overhead and profit. It was then marked down by 10% to move the stock. A week later the shopkeeper offered a further discount of 30% of the last marked price.

a. What was the final marked price (nearest dollar) of the toaster? 1 mark

b. A person bought the toaster and paid with a counterfeit $50 note. Calculate the loss to the shopkeeper, not counting the overhead costs. 1 mark

Question 4

A business manager investigates the depreciation of a new Hyundai i35 car costing $27000.

a. Calculate the book value of the car after 5 years if the reducing balance depreciation rate is $17\frac{1}{2}\%$ p.a. 1 mark

b. Determine the annual flat rate of depreciation to give the same book value at the end of 5 years as in part a. 1 mark

c. Determine the unit cost depreciation rate (i.e. $ per km, 4 decimal places) to give the same book value at the end of 5 years as in part a if the car is expected to travel an average distance of 500 km per week. 1 mark

Question 5

Jackie decides to buy an investment property costing $550000 in total. She has $50000 in savings and a bank is happy to loan her the balance. It is a 5 year term with monthly payments to cover interest only. She still owes the bank $500000 at the end of the 5 year term. The interest rate is 5.28% per annum.

a. Calculate the amount of the monthly payment Jackie has to make. 1 mark

b. What is the *effective* annual interest rate for the loan? 1 mark

At the end of the initial 5 year term, Jackie agrees to pay off the loan in 240 equal monthly installments. The interest rate is kept at 5.28% per annum. The balance of the loan amount is adjusted after each installment.

c. Calculate the amount of monthly installment. 1 mark

d. What is the *effective* annual interest rate over the loan period? 1 mark

e. Assuming CPI increases by an average rate of 4.0% annually over the 25 year loan period, what is the CPI adjusted value of Jackie's investment property at the end of the period? 1 mark

End of Exam 2

