2014 Further Mathematics Trial Exam 1 Solutions

SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
A	C	C	E	E	E	D	D	D	D	C	A	C

SECTION B
Module 1: Number patterns

1	2	3	4	5	6	7	8	9
A	A	B	C	B	D	B	D	D

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
A	B	B	B	E	C	C	E	E

Module 6: Matrices

1	2	3	4	5	6	7	8	9
D	A	C	C	D	C	A	A	B

SECTION A Core: Data analysis

Q1 The private sector had a greater increase in percentage and the actual amount. A

Q2 Let x be the percentage of employees in the public sector.
$1513.60x + 1396.60(1-x) = 1420.90$, $x \approx 21\%$ C

Q3 Compare the difference in full-time earnings and all employees earnings in both sectors. C

Q4 E

Q5 $Q_1 = 2.8$, $Q_3 = 4.5$, $IQR = 4.5 - 2.8 = 1.7$
$Q_1 - 1.5 \times IQR = 0.25$, $Q_3 + 1.5 \times IQR = 7.05$
$\therefore$ the outliers are 0, 0.1, 0.2 and 7.1. E

Q6 Victoria: $510.5 = \mu - \sigma$ $\therefore$ % of students less than 510.5
$= \dfrac{100-68}{2} = 16$, $\therefore$ % of students greater than $510.5 = 84$ E

Q7 84% were lower than z-score of 1. D

Q8 By CAS, $r = 0.8105$ D

Q9 $y = 158x - 38$, $x = 0.9$, $y = 104.2$
Residual $= 63 - 104.2 = -41.2$ D

Q10 D

Q11 $\dfrac{5.2+5.1+5.2+5.1+4.9+5.1+5.3+5.2+X+5.4+5.3+5.3}{12} = 5.1833$
$\therefore X = 5.0996 \approx 5.1$ C

Q12 Sum of seasonal indices $= 12$, $Y = 1.0225$ A

Q13 Seasonally adjusted rate $= \dfrac{5.8}{1.0287} \times 100\% \approx 5.6\%$ C

SECTION B
Module 1: Number patterns and applications

Q1 The nth term is $t_n = t_{n-1} - (2n-3)$ where $t_1 = 34$
$\therefore t_8 = -2 - 13 = -15$
$t_9 = -15 - 15 = -30$, $t_{10} = -30 - 17 = -47$ A

Q2 The sequence: 1, 7, 19, 37, ……
$t_n = t_{n-1} + 6(n-1)$ where $t_1 = 1$
$\therefore t_4 = 19 + 18 = 37$, $t_5 = 37 + 24 = 61$, $t_6 = 61 + 30 = 91$ A

Q3 $-7.1, -6.3, -5.5, \ldots\ldots, 7.7$ is an arithmetic sequence with a common difference of 0.8. $-5.5 + 7 \times 0.8 = 0.1$ B

Q4 $t_1 = -2$, $T_1 = (t_1)^2 = 4$
$t_2 = -2 + \dfrac{1}{2} = -\dfrac{3}{2}$, $T_2 = (t_2)^2 - \dfrac{1}{4} = 2$, $\therefore T_2 - T_1 = 2 - 4 = -2$ C

Q5 $t_n \times T_n = 1 \times (0.75)^{n-1}$ is a geometric sequence with $a = 1$ and $r = 0.75$. $S_\infty = \dfrac{1}{1-0.75} = 4$ B

Q6 $t_1 \times t_2 \times t_3 = (-2)(-2r)(-2r^2) = -8r^3 = -\dfrac{1}{8}$, $\therefore r = \dfrac{1}{4}$
$t_1 + t_2 + t_3 = (-2) + \left(-\dfrac{1}{2}\right) + \left(-\dfrac{1}{8}\right) = -\dfrac{21}{8}$ D

Q7 Let 1 unit be the amount at the end of the first half-year.
The amount at the end of the second half-year
$= 1\left(1 + \dfrac{r}{100}\right)^2 = 1 + \dfrac{2r}{100} + \left(\dfrac{r}{100}\right)^2$
Increase $= \dfrac{2r}{100} + \left(\dfrac{r}{100}\right)^2$
$\therefore$ % increase $= \left(\dfrac{2r}{100} + \left(\dfrac{r}{100}\right)^2\right) \times 100\% = \left(2r + \dfrac{r^2}{100}\right)\%$ B

Q8 $t_{n+1} + 2t_n = 2(t_{n+1} + 2.5t_n)$ and $t_{10} = -81$
$\therefore t_n = -\dfrac{1}{3}t_{n+1}$ and $t_{10} = -81$
$\therefore t_9 = -\dfrac{1}{3} \times -81 = 27$, $t_8 = -\dfrac{1}{3} \times 27 = -9$, $t_7 = -\dfrac{1}{3} \times -9 = 3$ D

Q9 $t_n = 5 \times 2^{n-1} - 1$, $\therefore t_{n+1} = 5 \times 2^n - 1 = 2t_n + 1$ D

Module 5: Networks and decision mathematics

Q1 In a complete graph of n vertices, every pair of vertices are adjacent, and the graph has $\frac{n(n-1)}{2}$ edges. A

Q2 Longest Hamiltonian circuit: E-A-F-G-D-B-C-E
$60+72+25+40+27+13+18=255$ B

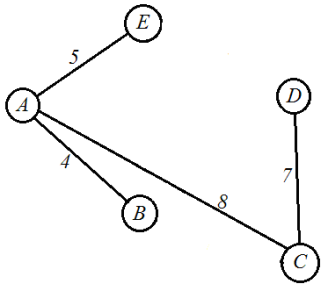
Q3 B

Q4 Minimum cut: $10+9+5+6=30$ B

Q5 All vertices have even degree, .: an Euler circuit exists and can start at any vertex. E

Q6 A tree graph does not have a circuit. C

Q7 $4000+5000+8000+7000=24000$ C



Q8 Critical path: *BDGJ*
Minimum time $=6+2+5+6=19$ E

Q9 Earliest start time for activity *E* is 5, latest start time is 9.
Float time $=9-5=4$ E

Module 6: Matrices

Q1 D

Q2 $\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a-c & \\ & \end{bmatrix}=\begin{bmatrix}0 & \\ & \end{bmatrix}$
$\therefore a=c$ A

Q3 X must be a 3×1 matrix. Let $X=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$.
$\begin{bmatrix}a\\ b\\ c\end{bmatrix}\begin{bmatrix}-1 & 1\end{bmatrix}=\begin{bmatrix}1 & -1\\ -1 & 1\\ 2 & -2\end{bmatrix}$, $\therefore \begin{bmatrix}-a & a\\ -b & b\\ -c & c\end{bmatrix}=\begin{bmatrix}1 & -1\\ -1 & 1\\ 2 & -2\end{bmatrix}$
$\therefore a+b+c=-2$ C

Q4 C

Q5 $y=\frac{-2z-13}{13}$ and $x=\frac{1}{2}(3y+5)$
When $z=-1$, $y=-\frac{11}{13}$ D

Q6 The transition matrix is $\begin{matrix}&N&S\\N\\S\end{matrix}\begin{bmatrix}0.99 & 0.05\\ 0.01 & 0.95\end{bmatrix}$
$\begin{bmatrix}0.99 & 0.05\\ 0.01 & 0.95\end{bmatrix}^2\begin{bmatrix}0.90\\ 0.10\end{bmatrix}\approx\begin{bmatrix}0.892\\ 0.108\end{bmatrix}$ C

Q7 $\begin{bmatrix}0.99 & 0.05\\ 0.01 & 0.95\end{bmatrix}^2\begin{bmatrix}\frac{x}{100}\\ 1-\frac{x}{100}\end{bmatrix}=\begin{bmatrix}0.90\\ 0.10\end{bmatrix}$
$\begin{bmatrix}\frac{x}{100}\\ 1-\frac{x}{100}\end{bmatrix}=\left(\begin{bmatrix}0.99 & 0.05\\ 0.01 & 0.95\end{bmatrix}^{-1}\right)^2\begin{bmatrix}0.90\\ 0.10\end{bmatrix}\approx\begin{bmatrix}0.91\\ 0.09\end{bmatrix}$, $\therefore x\approx91$ A

Q8 $\frac{0.01}{0.05+0.01}\approx0.17$ A

Q9 Mismatch of *c o p* and *c p o* in B B

$\begin{matrix}c & p & o\end{matrix}$
$\begin{bmatrix}1.00 & 0.95 & 1.50\\ 0.95 & 1.00 & 1.50\\ 1.00 & 0.85 & 1.55\\ 1.05 & 1.00 & 1.45\end{bmatrix}\begin{matrix}2\\3\\5\end{matrix}\begin{matrix}c\\o\\p\end{matrix}$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

