



Online & home tutors Registered business name: itute ABN: 96 297 924 083

Further Mathematics

2013

Trial Examination 2

Core – Data analysis

Module 2 – Geometry and trigonometry

Module 3 – Graphs and relations

Module 4 – Business-related mathematics

Instructions:

Answer all questions in the core and the three modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions.

Core – Data analysis

Question 1

The following table shows the top responses of the methods of travel to work in 2011 Census.

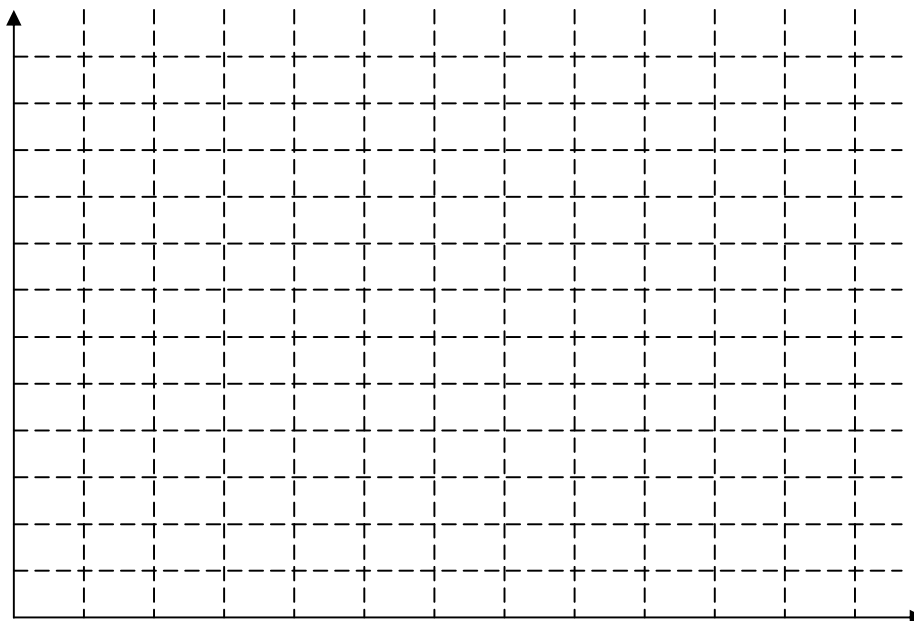
<u>Travel to work, top responses</u>	Greater Melbourne	%
<i>Employed people aged 15 years and over</i>		
Car, as driver	1,165,536	60.5
Train	115,979	6.0
Car, as passenger	83,808	4.3
Walked only	56,414	2.9
Tram	42,404	2.2

Source: abs

- a. What percentage of the employed people aged 15 years and over travel to work on public transport (train and tram) in Greater Melbourne on the day of 2011 Census? 1 mark

- b. Give a one sentence explanation why the percentages do not total to 100%. 1 mark

- c. Draw a bar chart showing the top second, third, fourth and fifth responses of the methods of travel to work in 2011 Census. 1 mark



Question 2

The following table shows the Australian population from 2000 to 2012.

<i>Year</i>	<i>Population</i>
2000	19 153 400
2001	19 413 200
2002	19 651 400
2003	19 895 400
2004	20 127 400
2005	20 394 800
2006	20 697 900
2007	21 015 900
2008	21 384 400
2009	21 778 800
2010	22 065 300
2011	22 323 900
2012	22 683 600

Source: abs

Let $time = 0$ be year 2000 and $population = 19.15$ millions, $time = 1$ be year 2001 and $population = 19.41$ millions etc.

- a. The equation of the least squares regression line for the data set is $population(\text{millions}) = a \times time + b$. Find the values of a and b , correct to the first decimal place.

2 marks

- b. What was the average annual increase in population between 2000 and 2012, correct to the nearest hundred thousand?

1 mark

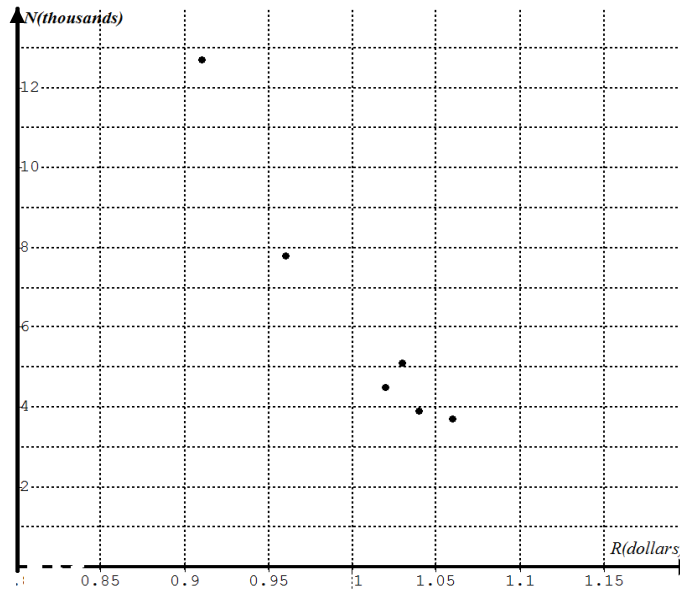
- c. Use the regression line to predict the Australian population in 2015.

1 mark

Question 3

The number of tourists N (in thousands) coming to Australia in the months January to June 2013 from a particular country varies with the A\$ to US\$ exchange rate R (in dollars). The relationship is shown in the following table and scatterplot. The relationship is non-linear.

	January	February	March	April	May	June
R	1.06	1.02	1.04	1.03	0.96	0.91
N	3.7	4.5	3.9	5.1	7.8	12.7



A reciprocal transformation applied to N can be used to linearise the scatterplot.

a. Apply this reciprocal transformation to the data and determine the equation of the least squares regression line for R and the reciprocal of N . Write the coefficients correct to three decimal places.

2 marks

b. Write down the value of Pearson's product moment correlation coefficient for the equation in **part a**, correct to three decimal places.

1 mark

c. Write down the proportion of the variation in the number of tourists that can be explained by the variation in the exchange rate in the model obtained in **part a**, correct to the nearest per cent.

1 mark

d. Predict the monthly number of tourists, correct to the nearest thousand, when the average exchange rate for the month is \$0.88.

1 mark

Question 4

The following table shows the rainfalls (mm) in summer, autumn, winter and spring for 2010, 2011 and 2012.

	summer	autumn	winter	spring
2010	188	253	259	265
2011	207	249	252	259
2012	179	257	263	271

- a. Using the data for the three years, determine the seasonal index for spring correct to three decimal places. 1 mark

- b. What does the seasonal index for spring tell us about the rainfall in spring? 1 mark

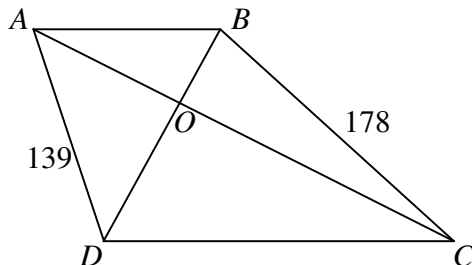
- c. Determine the deseasonalised spring rainfall in 2011. Write your answer correct to the nearest mm. 1 mark

Module 2: Geometry and trigonometry

Question 1

$ABCD$ is a trapezium in the following diagram. Side AB is parallel to side DC , and $DC = 2 \times AB$. AC and BD are perpendicular, and they intersect at O .

$AD = 139$ and $BC = 178$. All length measures are in metres.



a. Explain why $\triangle OAB$ is similar to $\triangle OCD$.

1 mark

Let $OA = a$ and $OB = b$.

bi. Show that $a^2 + 4b^2 = 19321$ and $4a^2 + b^2 = 31684$.

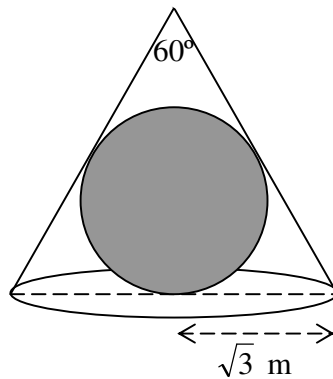
2 marks

bii. Hence find the length of side AB .

2 marks

Question 2

A cone made of sheet metal has a radius of $\sqrt{3}$ metres and a vertex angle of 60° . It is placed on a horizontal floor and completely enclosed a sphere.



a. Find the radius of the largest sphere possible.

1 mark

b. If the height and radius of the cone are both halved and the largest possible sphere is placed inside and completely enclosed by the cone, find the value of the ratio

volume of the second sphere : volume of the first sphere.

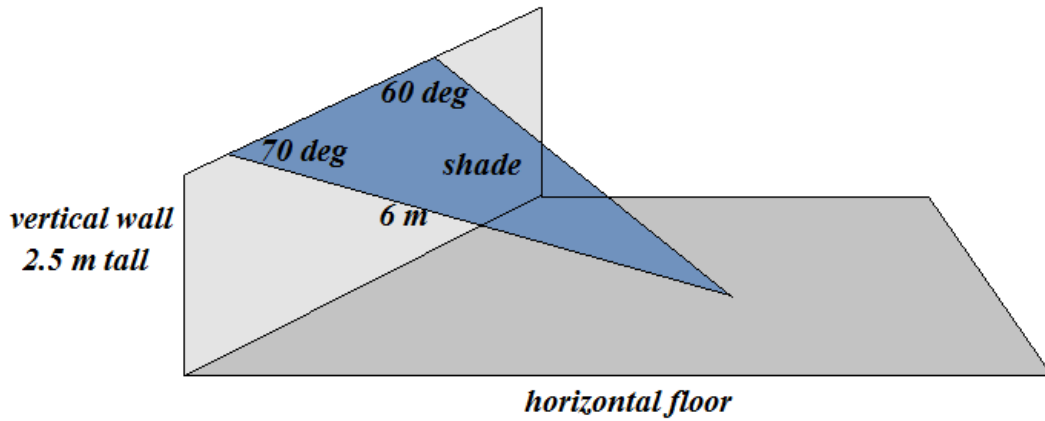
1 mark

c. If the height of the original cone in **part a** is doubled, find the magnitude of the vertex angle. Write your answer correct to the nearest degree.

1 mark

Question 3

One edge of a triangular shade is attached to the top of a 2.5 m tall wall and the opposite vertex is fastened to a horizontal floor. Some measurements of the shade are shown in the following diagram.



- a. Find the area of the shade. Write your answer correct to the nearest square metre. 2 marks

- b. Find the magnitude of the angle between the shade and the wall. Write your answer correct to the nearest degree. 1 mark

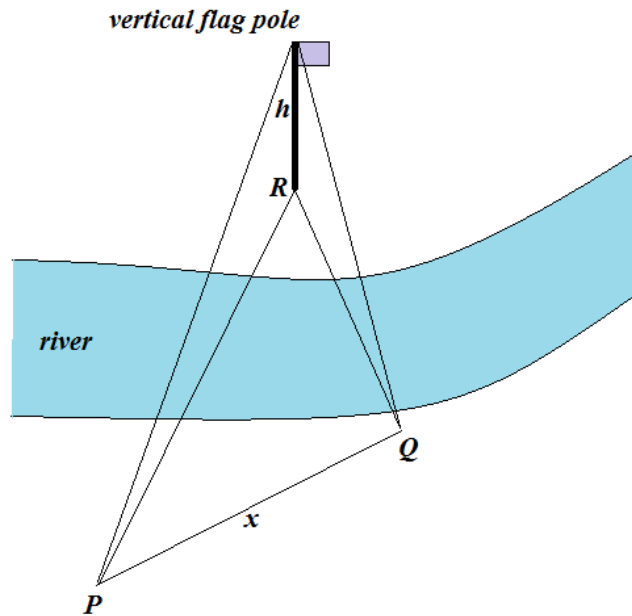
Question 4

In the following diagram P , Q and R are three locations 1.0 m above the water level of the river.

A h metre tall flag pole is located at R which is $N20^\circ E$ of P .

A student moves in a straight line x metres in the NE direction from P to Q .

At Q the bearing of R is $N25^\circ W$, and the angle of elevation of the top of the flag pole is 31°



a. Show that $QR \approx 0.598x$.

1 mark

b. Find the angle of elevation of the top of the flag pole when it is measured at P . Write your answer correct to the nearest degree.

2 marks

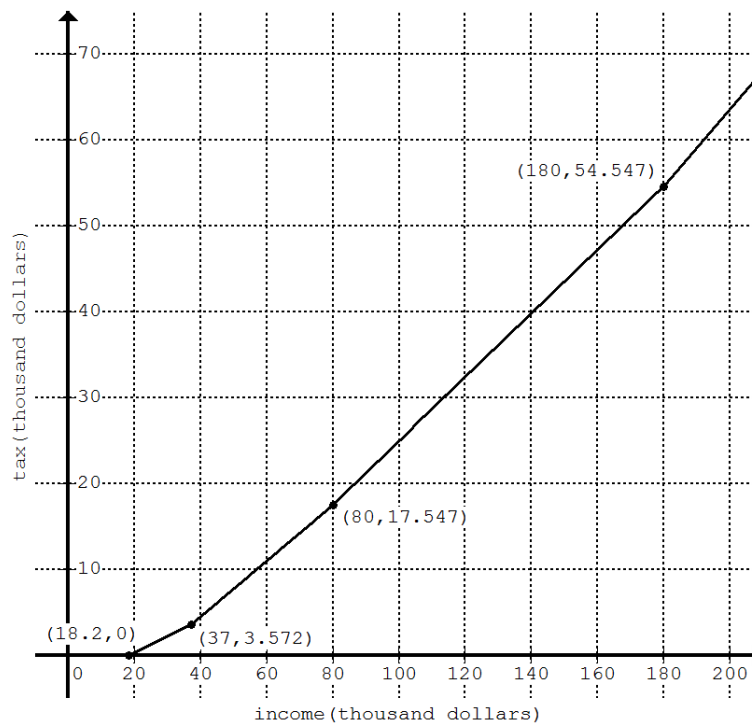
c. The distance x from P to Q is 30 metres. Find the height h of the flag pole. Write your answer correct to the nearest metre.

1 mark

Module 3: Graphs and relations

Question 1

The 2012/2013 Australian income tax (excluding Medicare Levy) for incomes up to \$210000 is shown in the following graph.



- a. No tax is payable if the income is \$ A or less. What is the value of A . 1 mark

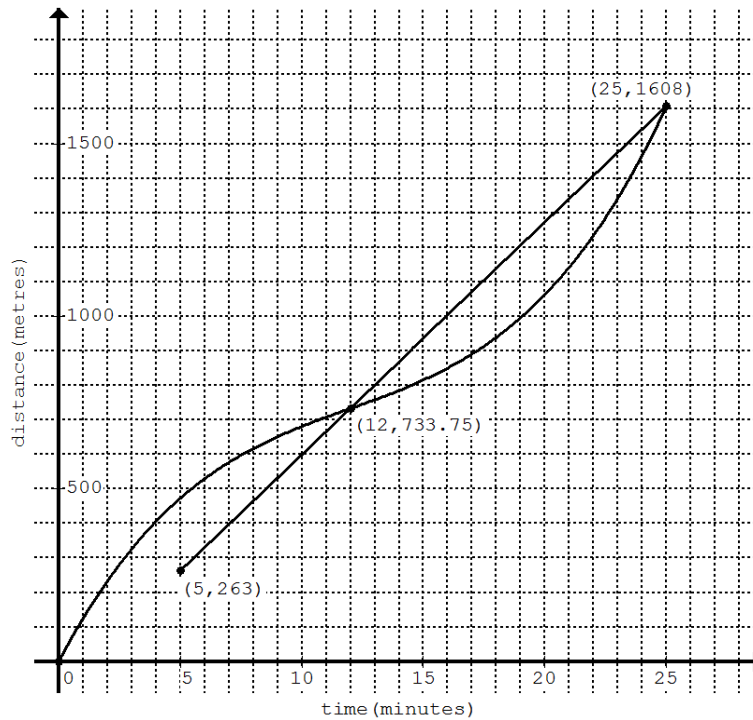
- b. What is the tax on each dollar of income above \$80000 up to \$180000? 1 mark

- c. Write an equation for calculating income tax \$ T for incomes \$ I between \$80000 and \$180000. 1 mark

- d. How much tax is payable on an income of \$87000? 1 mark

Question 2

Johnny and Jenny live in the same street. They walk to the train station from their homes. Jenny's home is 263 metres closer to the station than Johnny's home. Johnny starts walking at 12:00 midday ($time = 0$) and Jenny starts 5.0 minutes later. They both arrive at the train station at 12:25 pm. The distance-time graphs for Johnny and Jenny are shown below.



- a. How far is the train station from Jenny's home? 1 mark

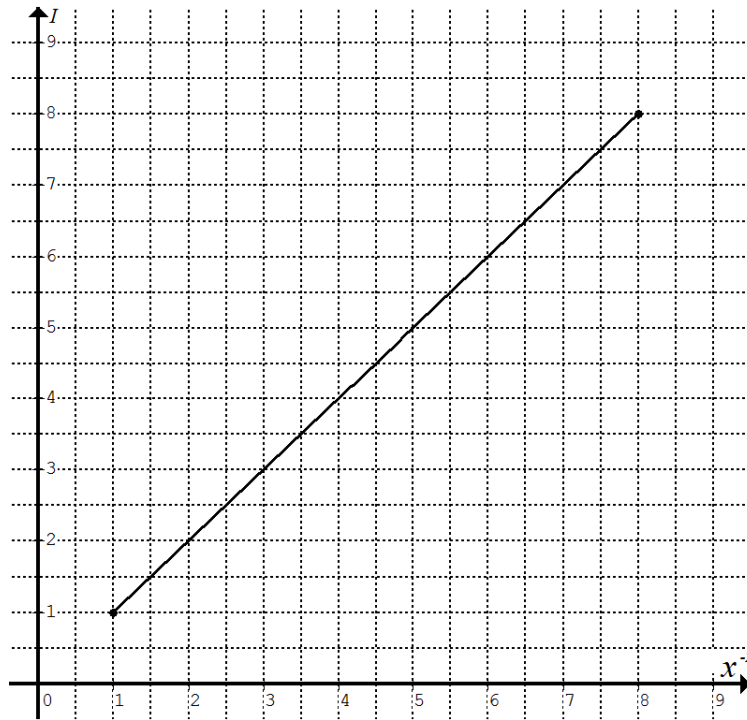
- b. Calculate the duration that Johnny is ahead of Jenny. Write your answer correct to the nearest minute. 1 mark

- c. At what time does Johnny *start* to speed up? 1 mark

- d. Calculate Johnny's average speed from his home to the train station. Write your answer correct to the nearest metre per minute. 1 mark

Question 3

The following graph shows that light intensity $I \propto x^{-2}$ for $1 \leq x^{-2} \leq 8$ where x is the distance from a light source. I is measured in lux and x is in metres.



a. Determine the equation relating I and x .

1 mark

b. Determine the distance from the light source when the light intensity is 6 lux. Write your answer correct to the nearest tenth of a metre.

1 mark

Question 4

A small manufacturer of handmade wooden toys employs two workers, Jill and Jack, to make rocking horses and dollhouses. The following table shows the time (hours) required by each worker to make an item. The maximum available time weekly of each worker is also shown.

	Jill	Jack
Rocking horse	1.8	1.4
Dollhouse	1.2	1.6
Maximum available time	38	45

The manufacturer makes \$35 per rocking horse and \$33 per dollhouse sold.

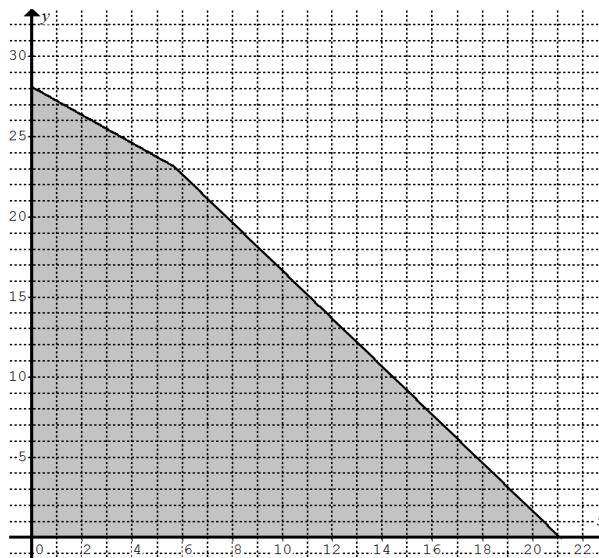
Let x be the number of rocking horses and y the number of dollhouses manufactured.

Note: x and y are whole numbers.

- a. Two of the constraints on x and y are represented by $x \geq 0$ and $y \geq 0$. Write two more inequations to represent the other constraints on x and y . 2 marks

- b. State the profit function $\$P$ when x rocking horses and y dollhouses are manufactured and sold. 1 mark

The feasible region for the values of x and y is shaded in the following graph.



- c. Determine the maximum weekly profit achievable by the manufacturer. 2 marks

Module 4: Business-related mathematics

Question 1 John invests \$25000 in a 6-month term deposit at 3.25% p.a. for a year. The interest received at the end of the first term is reinvested.

- a. Calculate the interest received at the end of the first term. 1 mark

- b. Calculate the value of John's investment at the end of the year. 1 mark

Question 2 A loan of \$350000 is taken out initially over 5 years at an interest rate of 6.50% per annum to purchase an investment property. Interest is calculated monthly, and monthly instalments are to be paid to cover the interest only. The loan is to be repaid in full in a lump sum or renegotiated at the end of the 5 years.

- a. Find the amount of the monthly instalment. 1 mark

- b. Interest rate is reduced to 6.00% at the start of the second year. Calculate the reduction in each of the monthly instalment. 1 mark

At the end of the initial 5 year term, the borrower renegotiates the loan at 4.5% p.a. interest calculated monthly and can afford to pay monthly instalment of \$2200.

- c. Determine the number of monthly instalments required to repay the loan in full. Write your answer correct to the nearest whole number. 1 mark

Question 3 50 year old Jack needs \$500000 to support his lifestyle when he retires at 65 years of age. At present Jack has \$120000 in his superannuation fund returning an average of 6.5% p.a. compounded monthly.

- a. Calculate Jack's monthly contribution required to meet his retirement needs. 1 mark

- b. If Jack contributes \$1600 monthly, at what age can he retires with \$500000 superannuation payout. 1 mark

Question 4 Jill needs \$1200 desperately. She borrows the money from a loan shark and agrees to repay in full including fees and charges in 6 equal monthly repayments. The loan shark's fees and charges are shown in the following tables.

Loan Fees and Charges	
Loan Establishment Fee	20% of principal
Monthly Loan Fee	4% of principal per month
Direct Debit Fee (Payable to the direct debit provider)	FREE

Fees that may apply	
Payment Dishonour Fee (Each time a direct debit or other payment is not made in full or at a time a repayment is rescheduled forward at your request. This fee includes our internal administrative costs and external third party costs.)	\$35.00
Default Fee (Daily recovery fee to cover costs of administering your account while your account remains in arrears)	\$7.00 per day

a. Calculate the amount of each monthly repayment. 1 mark

b. Calculate the annual percentage rate on the borrowing equivalent to the fees and charges charged by the loan shark. 1 mark

c. Calculate the total amount of extra fees payable if Jill is late to pay the fifth repayment for a week. 1 mark

Question 5 The following table shows the Australian CPI in June 2012 and 2013.

Time	CPI
June 2012	100.4
June 2013	102.8

- a. Calculate the percentage rise (correct to 2 decimal places) in the cost of living from June 2012 to June 2013. 1 mark
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- b. A family required \$2800 take home pay for the month June 2012 to cover all the expenses. Estimate the family's take home pay (nearest \$) in June 2013 required to maintain the same living standard as in June 2012. 1 mark
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Question 6 A company car is bought for \$28000.

- a. Use reducing balance depreciation method to find the depreciated value of the car after 3 years if the rate of depreciation is 20% p.a. 1 mark
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- b. Calculate the annual flat rate of depreciation, correct to the nearest percent, to give the same depreciated value of the car after 3 years as in part a. 1 mark
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- c. Assume that an average distance of 20000 km is driven per year. Calculate the depreciation per km of travel, correct to the nearest cent, to give the same depreciated value of the car after 3 years as in part a. 1 mark
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End of Exam 2