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2013 Mathematical Methods

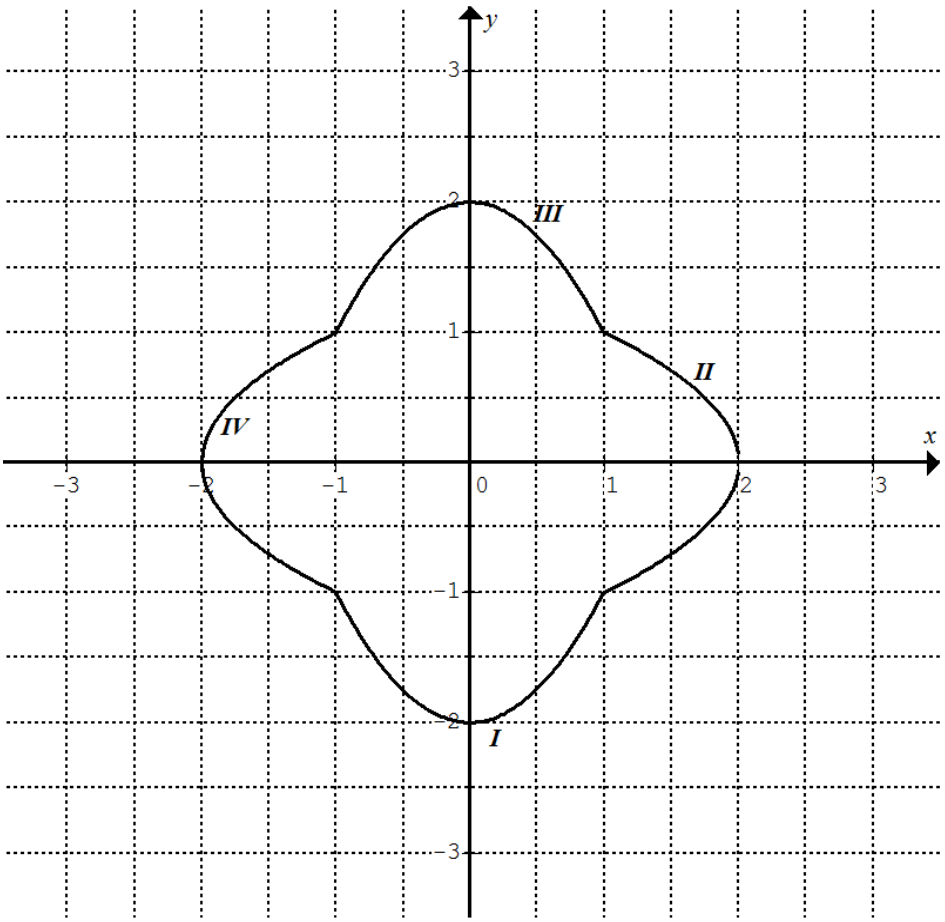
Year 12 Modelling Task

Time allowed: 2.25 hours

You are allowed: 1 bounded reference, 1 CAS, 1 scientific calculator
Working must be shown for questions worth 2 or more marks. Total: 80 marks

Theme: Designing a race circuit and a bike track

A teacher asks a student to design a race circuit by joining four identical ***parabolic*** sections *I*, *II*, *III* and *IV* in different orientations. Refer to the diagram below.
The four sections are joined at $(1,1)$, $(-1,1)$, $(-1,-1)$ and $(1,-1)$.
One of the four sections passes through $(0,-2)$.
All length measurements are in kilometres.



Question 1

a. Find the equation, domain and range of Section *I*. 4 marks

b. Write down the equation of Section *III*. 2 marks

c. Find the equations (y in terms of x) and domain of Section *IV*. 3 marks

d. Write down the equations (y in terms of x) for Section *II*. 2 marks

e. Find the gradient of Section *II* at $(1, -1)$. 3 marks

f. Explain why the four sections are not joined **smoothly** at $(1,1)$, $(-1,1)$, $(-1,-1)$ and $(1,-1)$.
Show calculations.

4 marks

In order to join Section *I* smoothly to Sections *II* and *IV*, the teacher makes the following suggestion:
Dilate Section *I* from the *x*-axis by a factor of *k* and then translate it in the negative *y*-direction by *c* units.

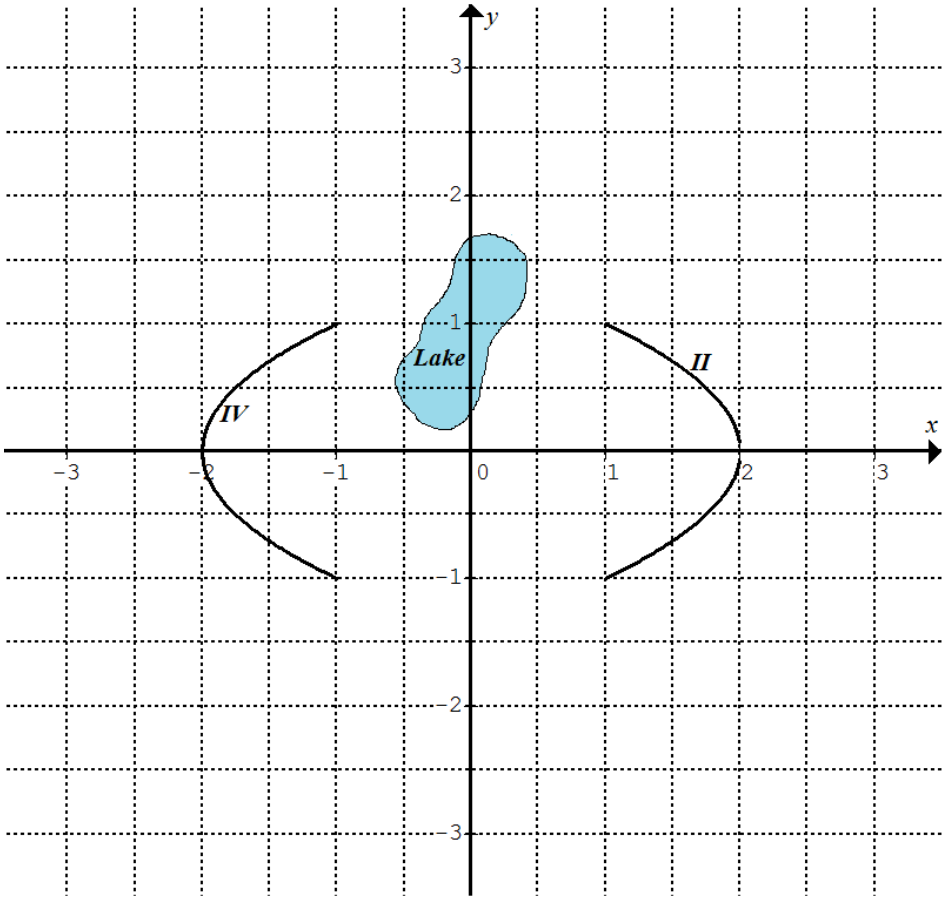
Question 2

a. Find the values of *k* and *c*, and write down the equation of the transformed Section *I*. 6 marks

b. Section *III* is also transformed so that it can join smoothly to Sections *II* and *IV*.
Write down the equation of the transformed Section *III*. 2 marks

c. Sketch accurately the new Section *I* and Section *III* suggested by the teacher on the diagram below. Label the turning points with coordinates.

4 marks



The new Section *III* crosses over the lake.

d. Apply a further transformation to each of the four sections of the new design suggested by the teacher that will avoid crossing the lake. Write down the equations (y in terms of x) of the four sections.

6 marks

To avoid crossing the lake the student suggests transforming Section *II* and Section *IV* instead, so that they join smoothly to the **original** Section *I* and Section *III*.

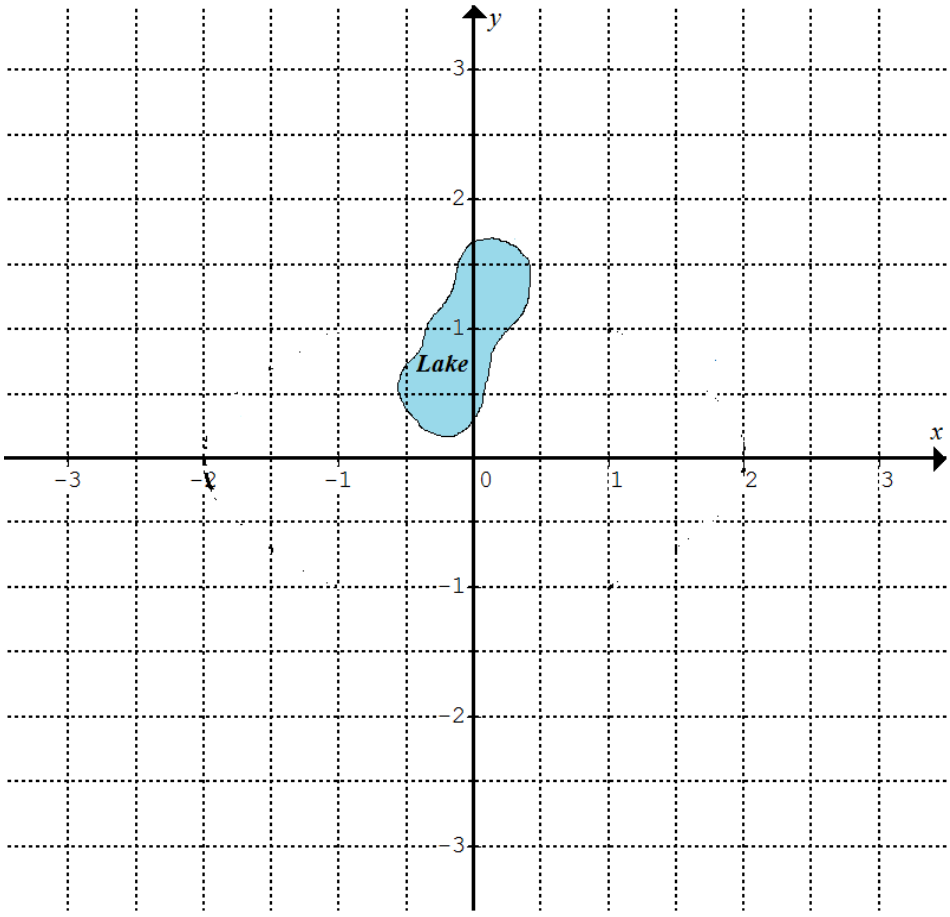
Question 3

a. Determine the equations (y in terms of x) of the transformed Section *II* and Section *IV* suggested by the student.

4 marks

b. Sketch the student's new design of the race circuit in the diagram below. Show and label the important points with their coordinates. Label each section with its equation.

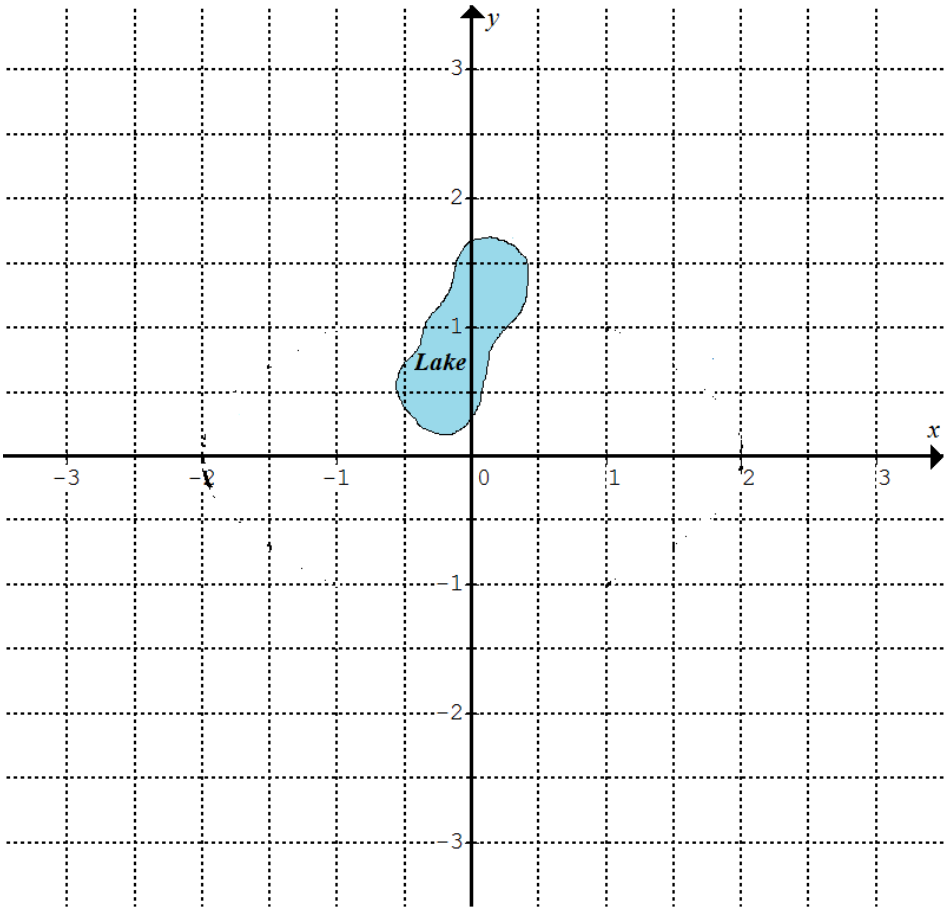
8 marks



Question 4 Design your own race circuit centred at the origin $(0,0)$. It must consist of four ***identical parabolic*** sections in different orientations joined smoothly together. The distance from the origin to any point on the race circuit must be between 1.5 km and 2 km inclusive.

8 marks

You need to show explanation/reasoning, working, equations and a labelled graph to score the 8 marks.



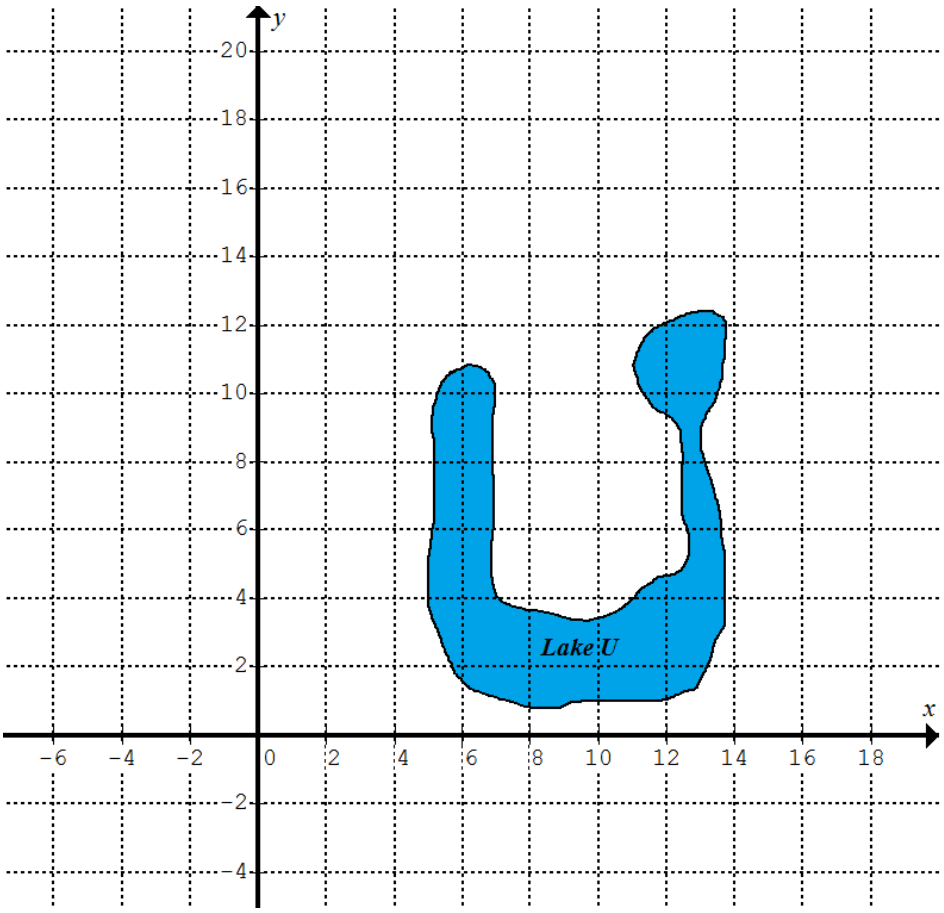
The teacher asks another student to design a bike track using quartic functions.

The ***first*** section of the bike track has equation $y = \frac{x^4}{16} - \frac{x^2}{2} + 1,\ -4 \le x \le 4$.

All length measures are in kilometres.

Question 5

a. Use CAS to sketch the section of the bike track in the diagram below.
Show the axis-intercepts, stationary points and end points with their coordinates. 5 marks



A ***second*** quartic section of the form $y = a(x-6)^4 + c,\ 4 \le x \le 8$, is connected smoothly to the ***first*** quartic section at $x = 4$.

b. Find the value of a and the value of c. 6 marks

c. Sketch accurately the ***second*** quartic section on the diagram in **Question 5** a.
Show the coordinates of the stationary point. 3 marks

The ***third*** section is also a quartic curve similar to the ***first*** section. It is the dilations of the ***first*** section by a factor of $\frac{1}{2}$ in both x and y directions. It is then translated in both directions by certain units so that its left endpoint joins the ***second*** section at $x = 8$.

d. State the domain and range of the ***third*** section. 2 marks

e. Show that the equation of the ***third*** section is $y = 0.5(x-10)^4 - (x-10)^2 + 5$. 4 marks

f. Show that the ***second*** and the ***third*** sections are joined smoothly at $x = 8$. 2 marks

g. Sketch accurately the ***third*** quartic section on the diagram in **Question 5** a.
Label the important points with their coordinates. 2 marks

End of task