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## 2012 VCAA Further Mathematics Exam 2 Solutions



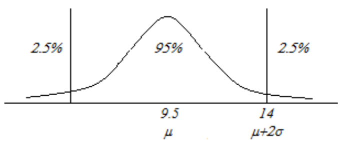
### Core – Data analysis

Q1ai There are 30 dots. The median is between the 15th and the 16th dots, i.e. $20°$ C

Q1aii There are 7 dots on the left of (less than) $16°$ C.

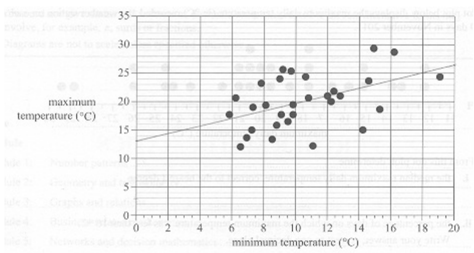
$\textit{Percentage of days} = \frac{7}{30} \times 100\% \approx 23.3\%$

Q1b $14 = 9.5 + 2 \times 2.25$, i.e. $\mu + 2\sigma$




$\textit{Percentage of days} \approx 95\% + 2.5\% = 97.5\%$

Q2a Two convenient points used to sketch the regression line are $(0,13)$ and $(20,26.4)$.




Q2b When the minimum temperature is $0°$ C, the maximum temperature is $13°$ C.

Q2c Given $r = 0.630$, strength: moderate; direction: positive

Q2d When the minimum temperature increases by $1°$ C, the maximum temperature increases by $0.67°$ C.

Q2e $r^2 = 0.630^2 \approx 0.40$, i.e. $40\%$

Q2f When the minimum temperature was $11.1°$ C,
$\textit{the predicted maximum temperature} = 13 + 0.67 \times 11.1 \approx 20.44°$
$\textit{Residual} = 12.2 - 20.44 \approx -8°$ C

Q3a [south-east] [north-east]

Q3b

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 3 | 4 | 4 | 4 | 4 |

↑ 2 ↑ 3.5 ↑ 4

Q4a By CAS, $(ws3.00pm)^2 = 3.4 + 6.6 \times ws9.00am$

Q4b When $ws9.00am = 24$, $(ws3.00pm)^2 \approx 161.8$, $ws3.00pm \approx 13$

### Module 1: Number patterns

Q1a $d = 162 - 168$ or $156 - 162 = -6$

Q1b $t_6 = 168 + (6-1) \times {}^-6 = 138$

Q1c There is a difference of 6 blocks from one month to the next, *the difference* $= 6 \times 2 = 12$ blocks

Q1di $S_{18} = \frac{18}{2}\left(2 \times 168 + (18-1) \times {}^-6\right) = 2106$ blocks

Q1dii The month ($n$) when $t_n = 0$, $168 + (n-1) \times {}^-6 = 0$, $n = 29$
*Total number of blocks in the estate*
$= S_{29} = \frac{29}{2}\left(2 \times 168 + (29-1) \times {}^-6\right) = 2436$
*Number of blocks left after 18 months* $= 2436 - 2106 = 330$

Q2a

| 1st year | 2nd year | 3rd year | |
|---|---|---|---|
| $t_1$ | $t_2$ | $t_3$ | |
| 16 | $16 \times 1.5$ | $16 \times 1.5^2$ | …… |

$t_3 = 16 \times 1.5^2 = 36$ building applications

Q2b Keep on multiplying by 1.5 until 100 is first exceeded, $t_6 = 16 \times 1.5^5 = 121.5$, i.e. the 6th year.

Q2c $S_5 = \frac{16(1.5^5 - 1)}{1.5 - 1} = 211$

Q2d By CAS, or repeating calculation with increasing $n$, $S_6$, $S_7$, ……, $S_8 \approx 788$, $S_9 \approx 1198$ .: the 9th year

Q2e $t_{n+1} = a \times t_n + b$, $t_1 = c$
Given that it forms a geometric sequence, .: $b = 0$ and $a = 1.5$.
$c = t_1 = 16$

Q3a $P_{n+1} = 0.96 \times P_n + 500$
$P_1 = 50$
$P_2 = 0.96 \times P_1 + 500 = 0.96 \times 50 + 500 = 548$
$P_3 = 0.96 \times P_2 + 500 = 0.96 \times 548 + 500 \approx 1026$, the population at the end of the third year is 1026.

Q3b By CAS, or repeating calculation with increasing $n$, $P_4$, $P_5$, ……, $P_{10} \approx 3878$, $P_{11} \approx 4223$, at the end of the 11th year.

Q3c At the end of the nth year, $P_n$
$= 50 \times 0.96^{n-1} + 500 + 500 \times 0.96 + 500 \times 0.96^2 + ... + 500 \times 0.96^{n-2}$
When $n$ is very large, $50 \times 0.96^{n-1}$ approaches 0, the remaining terms form an infinite geometric series $S_\infty = \frac{500}{1 - 0.96} = 12500$.
.: the greatest possible population is 12500 according to this mathematical model.

## Module 2: Geometry and trigonometry

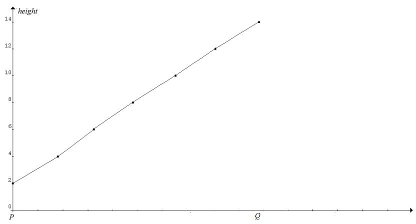
Q1a *Area of the rectangular block* $= 50\times 85 = 4250\ \text{m}^2$

Q1b *Volume of the prism* $= \frac{1}{2}\times(20\times 25\times 4) = 1000\ \text{m}^3$

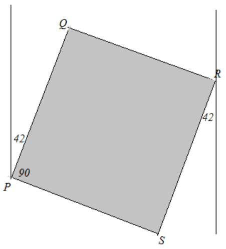
Q1c $AF = DE = \sqrt{4^2 + 25^2} \approx 25.32\ \text{m}$
*Total length of fencing* $= 25.32 + 20 + 25.32 + 20 \approx 90.6\ \text{m}$

Q2a *Difference in height* $= 14 - 2 = 12\ \text{m}$

Q2b




Q2ci *Bearing of S from P* $= 42° + 90° = 132°$

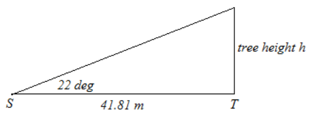



Q2cii *Bearing of S from R* $= 180° + 42° = 222°$

Q3a $\angle STP = 180° - (72° + 47°) = 61°$

The sine rule: $\frac{ST}{\sin 47°} = \frac{50}{\sin 61°}$, $ST \approx 41.81\ \text{m}$

Q3b




$\frac{h}{41.81} = \tan 22°$, $h = 41.81\times\tan 22° \approx 16.9\ \text{m}$

Q4a $\frac{OA}{10} = \cos 30°$, $OA = 10\times\cos 30° \approx 8.66\ \text{m}$

Q4b *Area of* $\Delta OAB = \frac{1}{2}\times 8.66\times 10\times\sin 30° \approx 21.7\ \text{m}^2$

Q4c $\frac{area.of\ \Delta OCD}{area.of\ \Delta OBC} = \left(\frac{OC}{OB}\right)^2$, $\frac{area.of\ \Delta OCD}{35} = \left(\frac{14}{10}\right)^2$
$area.of\ \Delta OCD = 35\times 1.4^2 \approx 68.6\ \text{m}^2$

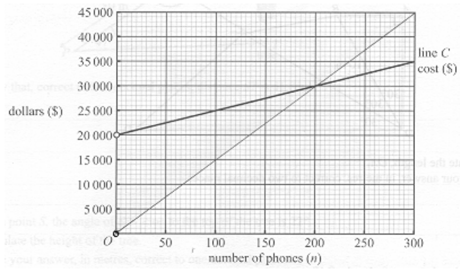
Q4d $BC = \sqrt{10^2 + 14^2 - 2\times 10\times 14\times\cos 30°} \approx 7.315\ \text{m}$
$\frac{\sin\angle BCO}{10} = \frac{\sin 30°}{7.315}$, $\sin\angle BCO \approx 0.6835$, $\angle BCO \approx 43°$
$\therefore\ \angle CDO = \angle BCO \approx 43°$

## Module 3: Graphs and relations

Q1ai *Gradient* $= \frac{35000 - 20000}{300 - 0} = 50$ dollars per phone

Q1aii $C = 20000 + 50n$

Q1bi




Note: Point $(0,0)$ is undefined.

Q1bii $R = 150n$, $54000 = 150n$, $n = 360$

Q1c To break even, $R = C$, $150n = 20000 + 50n$, $n = 200$

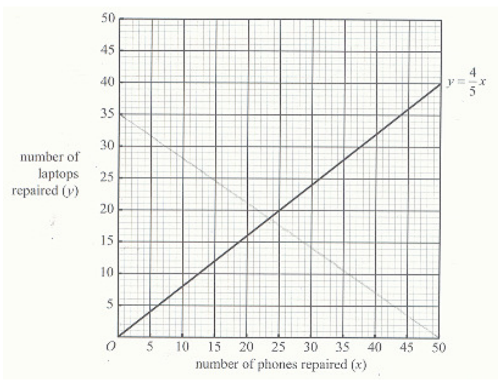
Q2a To obtain a profit, $R > C$, $600n > 320n + 125000$
$\therefore\ n > 446.43$, *minimum* $n = 447$

Q2b Let $\$p$ be the new selling price of each laptop.
To break even, $R = C$, $pn = 320n + 125000 + 50n$
To break even when $n = 400$,
$p\times 400 = 320\times 400 + 125000 + 50\times 400$, $p = 682.50$

Q3a The time available to repair *x* phones and *y* laptops is 1750 minutes.

Q3b $y \le \frac{4}{5}\times 10$, $\therefore\ y \le 8$, $\therefore$ 8

Q3c




Q3d Read from the graph, 18 laptops.

Q3e $35x + 50y \le 1750$, when $y = 9$, $x < 37.14$
$\therefore$ *maximum* $x = 37$

Q3fi *Profit* $= 60x + 100y$. To maximise the profit, repair 24 phones and 18 laptops (from graph).

Q3fii *Maximum profit* $= 60\times 24 + 100\times 18 = 3240$ dollars

**Module 4: Business-related mathematics**

Q1a *Deposit* $= \$8360 \times \frac{15}{100} = \$1254$

Q1bi *Amount owing* $= \$8360 - \$1254 = \$7106$

Q1bii Total interest paid $= \$650 \times 12 - \$7106 = \$694$

Q1c *Price before GST* $= \frac{\$8360}{1+\frac{10}{100}} = \$7600$

Q2a *Depreciated value* $= \$8360 - \$0.22 \times 3800 \times 3 = \$5852$

Q2b *Flat rate depreciation* $= \$8360 \times \frac{10}{100} = \$836$
*Unit cost depreciation* $= \$0.22 \times 3800 = \$836$, the same.

Q2c The equipment will be written off with a depreciated value of \$0 after $\frac{8360}{836} = 10$ years

Q2d *Depreciated value* $= \$8360 \times \left(1 - \frac{14}{100}\right)^{10} \approx \$1850$

Q3a Use CAS TVM Solver to obtain \$807.23

Q3b Use CAS TVM Solver to obtain 46.47, i.e. 47 months

Q3ci *Balance of the loan account* $= 40000 \times \left(1 + \frac{7.8}{12 \times 100}\right)^{12}$

Q3cii Monthly interest $= \$43234 \times \frac{7.8}{12 \times 100} \approx \$281.02$

Q4a Let $r$ be the annual interest rate.
$80000 \times \frac{r}{4 \times 100} = 1260$, $r = 6.3$

Q4b The investment amount remains the same, i.e. \$80000

Q4c Use CAS TVM Solver To obtain \$35208 approximately.
Set P/Y $= 4$ and C/Y $= 4$ for quarterly.

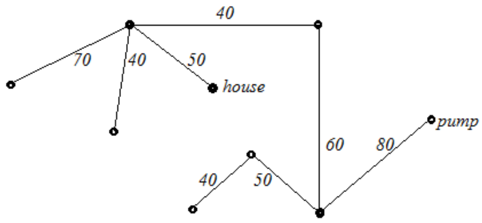
**Module 5: Networks and decision mathematics**

Q1ai $70 + 90 = 160$ m

Q1aii 2 vertices, the house and the top right location

Q1aiii An Eulerian path exists, starting from the house (odd degree) and finishing at the other odd-degree vertex with a distance of 1180 m. There is a another 70 m distance from this odd-degree vertex back to the house.
*Total distance* $= 1180 + 70 = 1250$ m

Q1bi




Q1bii Minimum spanning tree

Q2a Activity *E* can start after activities *A* and *D* are finished.
*Earliest start time for E* $= 10 + 2 = 12$ days

Q2b To indicate that activities *F*, *G* and *H* have the same earliest start time.

Q2c *Earliest start time for H* $= 10 + 5 = 15$ days

Q2d The critical path is *ABHILM*.

Q2e *The shortest time to complete all the activities* $= 10 + 5 + 4 + 3 + 4 + 2 = 28$ days
*Latest start time for activity J* $= 28 - 3 = 25$ days

Q3a 17

Q3b The minimum number of dashed lines must equal the number of tasks being allocated before allocation can be made.

Q3c

| | | | |
|---|---|---|---|
| 0 | 0 | 4 | 0 |
| 2 | 2 | 0 | 10 |
| 1 | 3 | 0 | 0 |
| 7 | 0 | 3 | 5 |

Q3d

| Worker | Task |
|---|---|
| Julia | *W* |
| Ken | *Z* |
| Lana | *X* |
| Max | *Y* |

## Module 6: Matrices

Q1a | Anvil | | Dantel |

Q1b $A \to B \to D \to C$

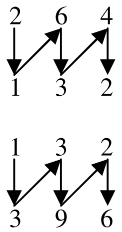
Q1c $G = KF = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 1 \end{bmatrix}$

Q1d Matrix G indicates the number of flights leaving each of the four cities,
e.g. *second column of* $G = 1\times1 + 1\times0 + 1\times0 + 1\times1 = 2$, i.e. there are two flights leaving Berga, one flies to Anvil and one to Dantel.

Q2ai

$$C = BA = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 4 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \end{bmatrix}$$

Q2aii The pattern is:




The disguise is 133926.

Q2b $A = B^{-1}C = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 1 & 3 & 2 \end{bmatrix}$

Q3a $70\% \times 100 + 80\% \times 200 + 90\% \times 50 = 275$

Q3b The staff will not return after leaving the industrial site.

Q3ci

$$S_{2012} = TS_{2011} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 70 \\ 170 \\ 65 \\ 45 \end{bmatrix}$$

Q3cii

$$S_{2013} = T^2 S_{2011} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix}^2 \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} = \begin{bmatrix} \\ 143 \\ \\ \\ \end{bmatrix}$$

The expected number of operators at the beginning of 2013 is 143.

Q3ciii

$$S_{2021} = T^{10} S_{2011} = \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix}^{10} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} \approx \begin{bmatrix} \\ 29.4 \\ \\ \\ \end{bmatrix}$$

.: year 2021

Q3civ

After many years (say 100), the state matrix becomes $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 350 \end{bmatrix}$.

.: *the total number of staff at the site* $= 0$

Q3d

$S_{2012} = TS_{2011} + A$

$$= \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \\ 50 \\ 0 \end{bmatrix} + \begin{bmatrix} 30 \\ 20 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 190 \\ 75 \\ 45 \end{bmatrix}$$

$S_{2013} = TS_{2012} + A$

$$= \begin{bmatrix} 0.70 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0 & 0 \\ 0 & 0.10 & 0.90 & 0 \\ 0.20 & 0.10 & 0.10 & 1.00 \end{bmatrix} \begin{bmatrix} 100 \\ 190 \\ 75 \\ 45 \end{bmatrix} + \begin{bmatrix} 30 \\ 20 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} \\ 182 \\ \\ \\ \end{bmatrix}$$

.: the expected number of operators at the beginning of 2013 is 182.

*Please inform mathline@itute.com re conceptual, mathematical and/or typing errors*