itute.com

Online & home tutors Registered business name: itute ABN: 96 297 924 083

Mathematical Methods(CAS)

2012

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1 The asymptote(s) of the graph of $f:[-4,4)\to R, f(x)=\dfrac{3}{(x+3)^2}+3$ is/are

A. $x=3$ only

B. $x=-3$ only

C. $x=3$ and $y=3$

D. $x=-3$ and $y=-3$

E. $x=-3$ and $y=3$

Question 2 $\log_{\frac{1}{e}}\left(\dfrac{1}{e^{\sqrt{x}}}\right)$ can be simplified to

A. $\sqrt{x}$

B. $\dfrac{1}{\sqrt{x}}$

C. $e^{1-\sqrt{x}}$

D. $e^{\sqrt{x}-1}$

E. $e^{1+\sqrt{x}}$

Question 3 The general solution to the equation $\sin(2x)+\cos(2x)=0$ is

A. $x=\dfrac{(2n+3)\pi}{8}$, $n\in Z$

B. $x=\dfrac{(2n-3)\pi}{8}$, $n\in Z$

C. $x=\dfrac{(4n-1)\pi}{8}$, $n\in Z$

D. $x=\left(n-\dfrac{1}{8}\right)\pi$, $n\in Z$

E. $x=\left(n+\dfrac{3}{8}\right)\pi$, $n\in Z$

Question 4 The maximal domain of $\{(x, y): y(x-2)=10, y \in [-5,0) \cup (2,\infty)\}$ is

A. $\left(-5,-\frac{10}{7}\right] \cup (0,\infty)$

B. $[-5,0) \cup (2,\infty)$

C. $(-\infty,0] \cup (2,7)$

D. $(-\infty,-1) \cup (2,6]$

E. $R \setminus \{2\}$

Question 5 Given $f:(-a,0] \to R, f(x) = -\sqrt{a^2-x^2}$ where $a \in R^+$, the intersection of the graphs of f and f^{-1} is defined for $x \in$

A. $(-a,0)$

B. $[-a,0]$

C. $(-a,0]$

D. $[0,a)$

E. $(0,a)$

Question 6 Given $f(x) = 2(x-a)^2(x-b)$ where $a > b > 0$, and $g(x) = e^x f(2x-c)$, $g(x) = 0$ will have two unique ***positive*** solutions if

A. $c > -b$

B. $c > -a$

C. $-a < c < -b$

D. $c < -b$

E. $c < -a$

Question 7 In the order shown, a translation of b units in the negative x-direction and a dilation parallel to the x-axis by a factor of 2 are applied to function $f(x) = 2(x+b)^3 - a$ to form function $g(x)$. $g(x) =$

A. $\frac{1}{4}(x+2b)^3 - a$

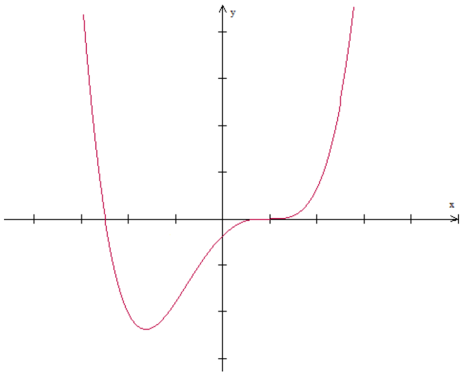
B. $2\left(\frac{x}{2}+b\right)^3 - a$

C. $\frac{1}{4}x^3 - a$

D. $2\left(\frac{x}{2}-b\right)^3 - a$

E. $\frac{1}{4}(x+4b)^3 - a$

Question 8 The graph of gradient function $f'(x)$ is shown below:



The stationary points of function $f(x)$ consist of

A. a local minimum point and a point of inflection

B. a local maximum point and a point of inflection

C. two local maximum points

D. two local minimum points

E. a local maximum point and a local minimum point

Question 9 Given $f(x) = e - \log_e(\log_e x)$, the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect at $y =$

A. e^e

B. e

C. 2

D. $0.5e$

E. 1

Question 10 If $\frac{1}{\sqrt{2}} \le \sqrt{1-\cos(2x)} \le 1$, the maximum range of values of x in the interval $\left[0, \frac{\pi}{2}\right]$ is

A. $\frac{\pi}{6} \le x \le \frac{\pi}{3}$

B. $\frac{\pi}{4} \le x \le \frac{\pi}{2}$

C. $\frac{\pi}{6} \le x \le \frac{\pi}{4}$

D. $\frac{\pi}{3} \le x \le \frac{\pi}{2}$

E. $0 \le x \le \frac{\pi}{2}$

Question 11 For $a > 0$, $f(x) = ae^{|a-x|}$ is a decreasing function in the interval

A. $\left[\frac{a}{2}, a\right)$

B. $\left(\frac{a}{2}, a\right]$

C. $(-\infty, a]$

D. $(a, 2a)$

E. (a, ∞)

Question 12 If $f(a) = b$, $f'(a) = \frac{1}{a}$ and $g(x) = f^{-1}(x)$, then $g'(b) =$

A. a

B. b

C. 1

D. $\frac{1}{b}$

E. $\frac{1}{a}$

Question 13 If $f'(x) = 0$ at points $(-1,1)$, $(1,2)$ and $(3,-1)$, then $f'(|x|) = 0$ at

A. $(-1,2)$, $(-3,-1)$, $(1,-2)$ and $(3,1)$

B. $(-1,-2)$, $(-3,1)$, $(1,2)$ and $(3,-1)$

C. $(-1,2)$, $(-3,-1)$, $(1,2)$ and $(3,-1)$

D. $(-1,2)$, $(-3,-1)$ and $(1,1)$

E. $(-1,2)$, $(-3,-1)$ and $(1,-1)$

Question 14 Given $\int_a^b f(x)dx = c$, the value of $\int_{\frac{a-h}{2}}^{\frac{b-h}{2}} 2f(2x+h)dx$ is

A. $0.25c$

B. $0.5c$

C. c

D. $2c$

E. $c-2h$

Question 15 Function g is defined as $g(x) = \tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$. The average value of g in the interval $\left[0, \frac{\pi}{8}\right]$ is closest to

A. $\frac{2\pi}{5}$

B. $\frac{60}{49}$

C. $\frac{\pi}{2}$

D. $\frac{6}{5}$

E. $\frac{\pi}{3}$

Question 16 If $f'(x) = \frac{1}{\sqrt{x+10}}$ and x changes from a to $1.02a$, by linear approximation y changes from $f(a)$ to $bf(a)$. Given $f(0) = 2\sqrt{10}$, the value of b is approximately

A. $\frac{1}{50\sqrt{a+10}}$

B. $\frac{a}{50\sqrt{a+10}}$

C. $\frac{a}{100\sqrt{a+10}}$

D. $\frac{1.02a+10}{a+10}$

E. $\frac{1.01a+10}{a+10}$

Question 17 Jack, Jill and four other people sit randomly in a row. The probability that Jack and Jill are next to each other is

A. $\frac{11}{24}$

B. $\frac{5}{12}$

C. $\frac{3}{8}$

D. $\frac{1}{3}$

E. $\frac{1}{4}$

Question 18 One face of a small cube is red, two faces are blue and the remaining three faces are green. The cube is rolled twice. The probability of getting one green and one blue in the two rolls, and the probability of getting green in the second roll if the first roll was blue are respectively

A. $\frac{1}{3}$ and $\frac{1}{3}$

B. $\frac{1}{3}$ and $\frac{1}{2}$

C. $\frac{1}{6}$ and $\frac{1}{3}$

D. $\frac{1}{6}$ and $\frac{1}{2}$

E. $\frac{1}{2}$ and $\frac{1}{2}$

Question 19 A ***biased*** coin is tossed n times and the number of tails is recorded. This procedure is repeated many times. The long-run average number of tails is 12.3, and 2.8 is the standard deviation. The value of n is closest to

A. 15

B. 24

C. 25

D. 34

E. 41

Question 20 $BAABABBA$ is a sequence of a two-state Markov chain.

If $\Pr(BAABABBA) = 1 \times \frac{1}{3} \times \frac{1}{4} \times ...... = \frac{1}{288}$, then the value of $\Pr(BBAABAB)$ is

A. $\frac{1}{64}$

B. $\frac{1}{96}$

C. $\frac{1}{128}$

D. $\frac{1}{192}$

E. $\frac{1}{256}$

Question 21 Random variable X has a normal distribution with μ and σ as its mean and standard deviation respectively. Given $\Pr(X > 12.5) = 0.8$ and $\Pr(X > 18.5 \mid X > 12.5) = 0.8$, which one of the following approximations is true?

A. $\mu - 2\sigma \approx 18.5$

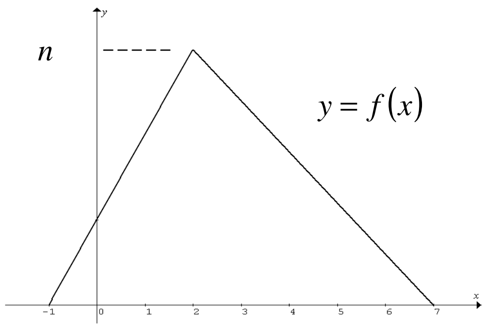
B. $\mu + 0.64\sigma \approx 18.5$

C. $\mu - 0.64\sigma \approx 18.5$

D. $\mu + 0.36\sigma \approx 18.5$

E. $\mu - 0.36\sigma \approx 18.5$

Question 22 The probability distribution for random variable X is shown below.



Which one of the following statements is ***NOT*** true?

A. $\int_{-2}^{8} f(x)dx = 1$

B. $n = 0.25$

C. $\Pr(2) = n$

D. The median of X is greater than the mode of X.

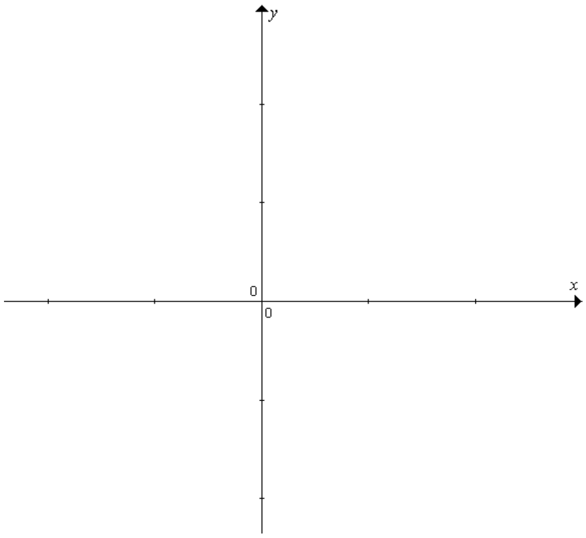
E. The mean of X is greater than the median of X.

Instructions for Section 2

Answer **all** questions.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 Function $f(x)$ is a *degree five* polynomial and $x \in (-1, 2]$. $f(x)$ has an inflection point at $x = 0$, and $f(0) = f'(0) = 0$. $f(x)$ has another inflection point at $x = 1$ where $f(1) = 1$ and $f'(1) = 0$.

a. Sketch a graph of $y = f(x)$ based on the given conditions. 2 marks



b. Given $f'(x) = ax^2(x-1)^2$ where $a \in R$.
Show $f'(x)$ satisfies the conditions given at the beginning of **Question 1**. 2 marks

c i. Find $f(x)$ in terms of a. 2 marks c ii. Hence find a. 1 mark

d. Find the coordinates of the point on the curve $y = f(x)$ in the interval $[0,1]$ such that the rate of change of y with respect to x is the greatest. 2 marks

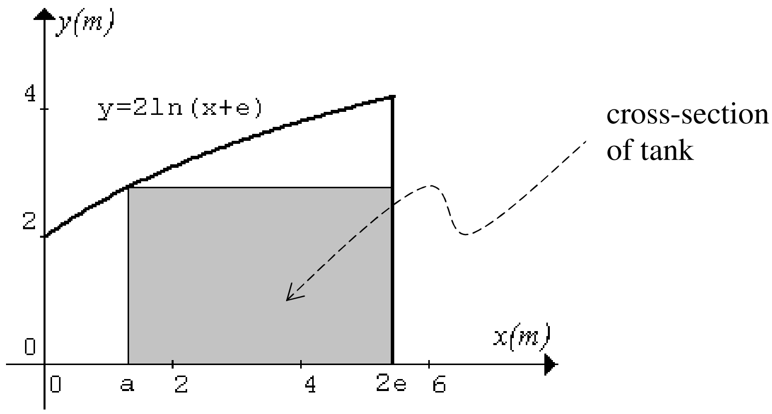
e. What is the total number of inflection points of $f(x)$? 1 mark

f. Determine the coordinates of the stationary inflection points of $g(x) = -2f(-x) + 2$. 2 marks

g. Determine the domain and range of $g(x)$. 3 marks

Question 2 A *8-metre* long rectangular tank of maximum volume is to be installed inside a tunnel of uniform cross-section as shown below. The cross-section of the tunnel is enclosed by $y = 2\log_e(x+e)$, the x-axis and $x = 2e$. The base of the tank lies between $x = a$ and $x = 2e$. The top corner at $x = a$ touches the ceiling of the tunnel.

Ignore the thickness of the tank walls.

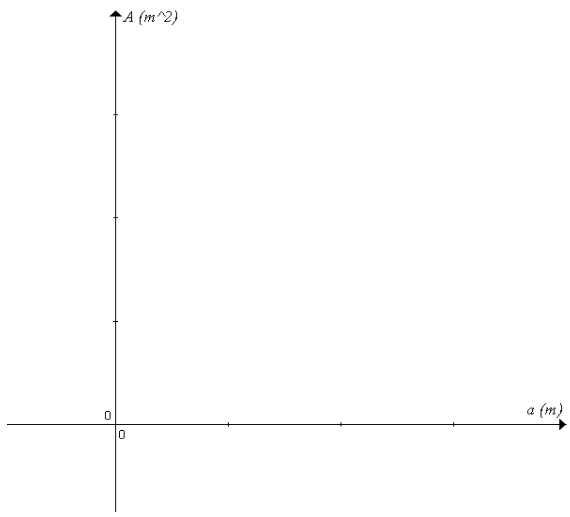


a. In terms of a find the cross-sectional area A of the tank. 1 mark

b i. ***Use calculus*** to find $\frac{dA}{da}$. 1 mark

b ii. Hence show that the volume of the tank is at its maximum value when its height is $\frac{2(2e-a)}{e+a}$.
1 mark

b iii Sketch the graph of A versus a. Show the important features. 3 marks



Water is pumped into the tank at $(90-36h)$ cubic metres per hour where h metres is the depth of water in the tank.

c. Determine the rate (2 decimal places, in metres per hour) at which the depth increases when $h=1$.
2 marks

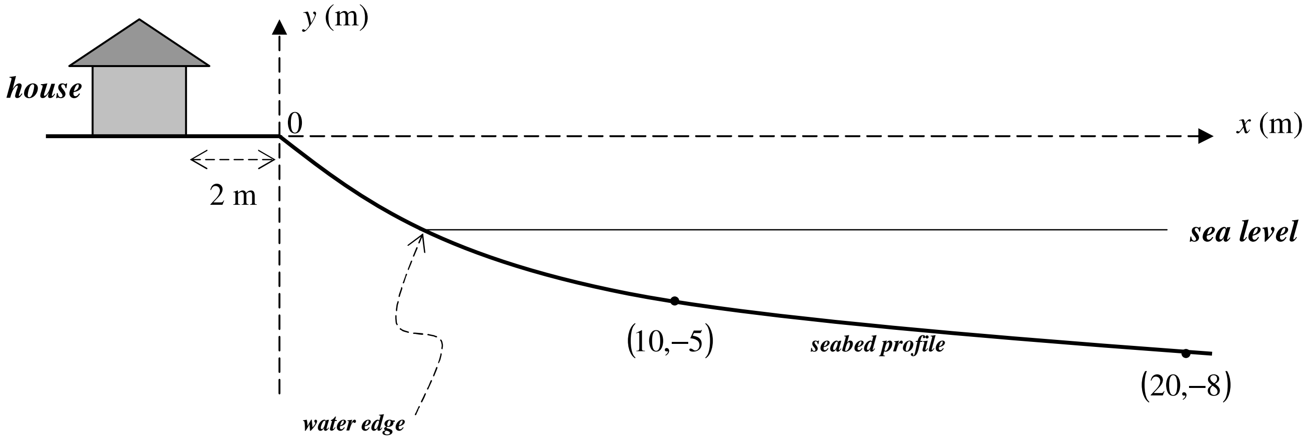
d. Given at time $t=0$, $h=0$, and $\frac{dt}{dh}=\frac{1}{2.5-h}$ where t is in hours, ***use calculus*** to find the exact value of t when the tank is filled to a depth of 1.25 metres.
3 marks

e. Calculate the maximum volume (nearest cubic metre) of *water* in the tank. 1 mark

Soil is needed to fill the space above the top and on the left side of the tank.

f. Determine the volume (nearest cubic metre) of soil required. 2 marks

Question 3



The seabed profile is given by $y = ae^{bx} + c$ and it passes through points $(0,0)$, $(10,-5)$ and $(20,-8)$.

a i. Show that $a = \frac{25}{2}$, $b = \frac{1}{10}\log_e\left(\frac{3}{5}\right)$ and $c = -\frac{25}{2}$. 4 marks

a ii. ***Use calculus*** to show $\frac{dy}{dx} = \left[\frac{5}{4}\log_e\left(\frac{3}{5}\right)\right]\left(\frac{3}{5}\right)^{\frac{x}{10}}$ 2 marks a iii. Evaluate the gradient (2 decimal places) of the seabed profile at $x = 10$. 1 mark

The changes in the sea level is given by $y = 3\sin\left(\frac{\pi t}{6}\right) - 5$, and $t = 0$ at 12:00 midday.

b i. How far below the house is the sea level at 6:00 pm? 1 mark

b ii. Determine the horizontal distance between the house and the water edge at 6:00 pm. 1 mark

c. Find the exact rate of change (metres per hour) of the sea level at 6:00 pm. 2 marks

d. Find the exact *horizontal* rate (metres per hour) that the water edge recedes from the house at 6:00 pm. 3 marks

Question 4 The Australian Egg Corporation is responsible for defining the following sizes of egg.

Size	**Weight** w (grams)
7	$w < 45$
6	$45 \le w < 50$
5	$50 \le w < 55$
4	$55 \le w < 60$
3	$60 \le w < 65$
2	$65 \le w < 70$
1	$70 \le w < 75$
0	$w \ge 75$

Eggs are sold in Victoria in the following standard sizes only.

Size	**Weight** w (grams)
large	$52 \le w < 60$
extra large	$60 \le w < 68$
jumbo	$w \ge 68$

An egg farm, Awesome Eggs, supplies eggs to Victoria only. The size of eggs produced by Awesome Eggs has a normal distribution with $\mu = 65$ and $\sigma = 8$.

a. Determine the proportion (4 decimal places) of eggs produced by Awesome Eggs meeting the Victorian standards.

1 mark

b. Determine the proportion (4 decimal places) of the size 0 to 6 (inclusive) eggs produced by Awesome Eggs meeting the Victorian standards.

1 mark

c. Awesome Eggs supplies all the standard-size eggs produced to individual food stores. The price list is shown in the table.

Size	**Price** ($ per dozen)
large	1.00
extra large	1.10
jumbo	1.20

Calculate the average price per dozen (nearest cent) of eggs produced by Awesome Eggs. 2 marks

d. From a box (a dozen) of Awesome extra large eggs what is the probability (2 decimal places) that it contains at least 6 eggs of size $65 \le w < 68$?

2 marks

e. Four eggs are taken randomly from a box (a dozen) of Awesome extra large eggs. What is the probability (2 decimal places) that the total weight of the four eggs is less than 250 grams?

2 marks

f. Given the general normal probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, find the mean weight (nearest gram) of a dozen of jumbo eggs from Awesome Eggs.

3 marks

g. Another egg farm, Best Eggs, also supplies eggs to Victoria only. The size of eggs produced by Best Eggs also has a normal distribution. 5% of its eggs are size 7 and 10% are size 0.
Find the mean and standard deviation (2 decimal places) of the size of eggs produced by Best Eggs.

2 marks

h. A food store owner buys eggs from Awesome Eggs or Best Eggs only. If he buys from Awesome Eggs one day, the probability of buying from Awesome Eggs again the next day is 0.36. If he buys from Best Eggs one day, the probability of buying from Best Eggs again the next day is 0.45. Today he buys eggs from Awesome Eggs. Find the probability (2 decimal places) that he buys eggs from Best Eggs in five out of the next six days.

2 marks

End of exam 2

