2012 Further Mathematics Trial Exam 2 Solutions

Core – Data analysis

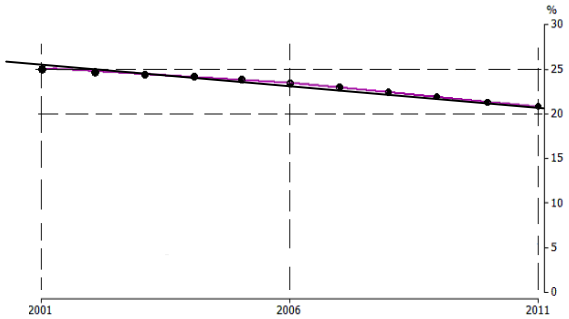
Q1a 20% had at least one overseas-born parent, .: 80% had both parents born in Australia.

Q1b 49 was the median age of the overseas-born population, .: 50% were under 49 in 2011.

Q1c *Male*: *female ratio* was $96.1:100$.

$$\textit{Female percentage} = \frac{100}{96.1+100}\times 100\% \approx 51\%$$

Q2a



Q2b When $t = 11$, $P = 21$

$$\textit{Gradient of the line of best fit} \approx \frac{21-24}{11-4} \approx -0.43$$

$$P = -0.43t + 25.73$$

Q2c When $t = 15$ (2015), $P = -0.43\times 15 + 25.73 \approx 19$

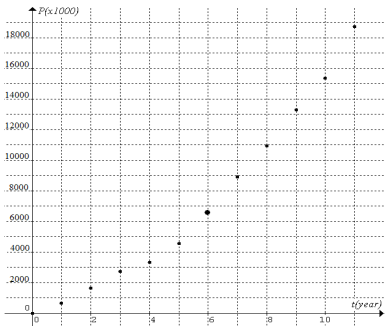
Q3a $\textit{Increase} \approx (8+7.9)\times 100000 - (4+4.1)\times 100000$
$= 780000$

Q3b There was a large increase in the number of young immigrants between 1991 and 2011.

Q3c The population structure changed from inverted Martini glass shape to inverted wine glass shape.

Q3d 25-29 age group

Q4a



Q4b $P = 150t^2 + 900$

Q4c When $t = 12$ (2021),
$P = 150\times 12^2 + 900 = 22500$ thousands
2021 population $= 3770 + 22500 = 26270$ thousands
≈ 26 millions

Module 2: Geometry and trigonometry

Q1a $\dfrac{H-h}{h} = \dfrac{\overline{AB}}{\overline{DC}} = \dfrac{3}{5}$, $\dfrac{H}{h} - 1 = \dfrac{3}{5}$, $\dfrac{H}{h} = \dfrac{8}{5}$, $\dfrac{h}{H} = \dfrac{5}{8}$

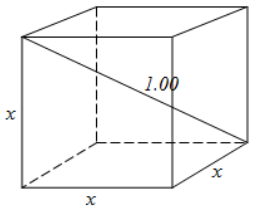
Q1b $\dfrac{\textit{area of }\Delta ABX}{\textit{area of }\Delta CDX} = \left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$

Q1c $\dfrac{\textit{area of }\Delta CDB}{\textit{area of }\Delta CDX} = \dfrac{\frac{1}{2}\overline{CD}\times H}{\frac{1}{2}\overline{CD}\times h} = \dfrac{8}{5}$

$\therefore \dfrac{\textit{area of }\Delta CDX + \textit{area of }\Delta BCX}{\textit{area of }\Delta CDX} = \dfrac{8}{5}$

$1 + \dfrac{\textit{area of }\Delta BCX}{\textit{area of }\Delta CDX} = \dfrac{8}{5}$, $\therefore \dfrac{\textit{area of }\Delta BCX}{\textit{area of }\Delta CDX} = \dfrac{3}{5}$

Q2a Let x m be the edge length of the largest cube.



$x^2 + x^2 + x^2 = 1.0$,

$x^2 = \dfrac{1.00}{3}$, $x = \sqrt{\dfrac{1.00}{3}}$

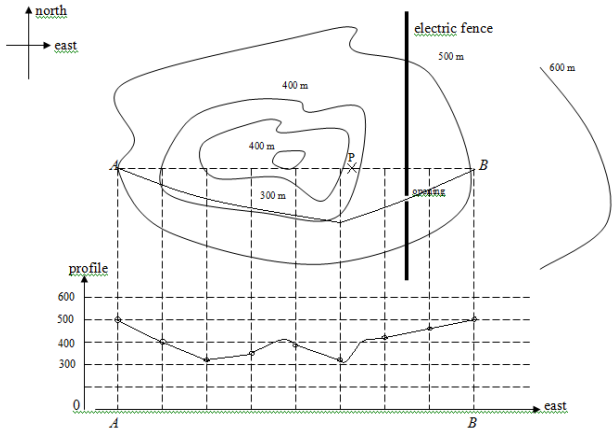
$\textit{Volume} = x^3 = \left(\sqrt{\dfrac{1.00}{3}}\right)^3 \approx 0.19\text{ m}^3$

Q2b $\textit{Surface area of the sphere} = 4\pi\left(0.50^2\right) = \pi$

$\textit{Surface area of the largest cube} = 6x^2 = 6\times\dfrac{1.00}{3} = 2$

Value of the ratio is $\dfrac{\pi}{2}$.

Q3abd



Q3c By estimation, gradient $\approx \dfrac{100}{80} = 1.25$

Q4a $\angle DAE = 60°$, $\dfrac{\sin\angle ADE}{400} = \dfrac{\sin 60°}{350}$, $\angle ADE \approx 82°$

$\therefore\ \angle CDE = 180° - 82° = 98°$

Q4b $\overline{BC} = x$, $\therefore\ \overline{DA} = x - 200$

Apply the cosine rule to ΔADE,

$350^2 = 400^2 + (x-200)^2 - 2(400)(x-200)\cos 60°$

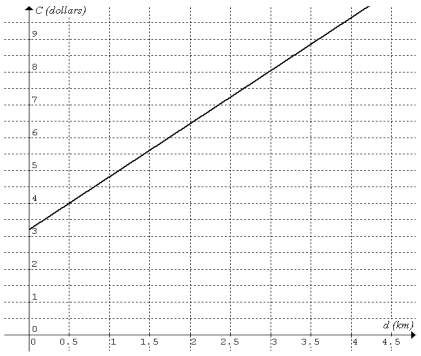
$\therefore\ x^2 - 800x + 157500 = 0$

Q4c *Area of* $\Delta ABC = \dfrac{1}{2}(450)(450\sin 60°) \approx 87685\ \text{m}^2$

Module 3: Graphs and relations

Q1a $C = 1.617d + 3.20$

Q1b

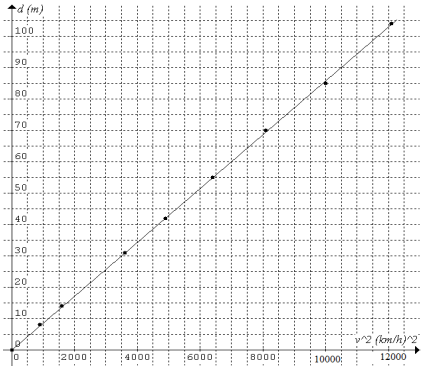


Q1c $51.70 = 1.617d + 3.20$, $d \approx 30$ km

Q1d $1.617d + 3.20 = 1.59d + 3.90$, $d \approx 26$ km

Q2a

v^2	900	1600	3600	4900	6400	8100	10000	12100
d (m)	8	14	31	42	55	70	85	104



Q2b $k \approx \dfrac{104}{110^2} \approx 0.0086\ \text{m h}^2/\text{km}^2$

Q2c $d \approx 0.0086v^2$, when $v = 55$ km/h, $d \approx 26$ m

Q3a 90 m

Q3b *Total distance* $= 90 + 55 = 145$ m

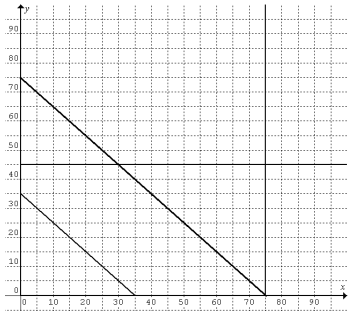
Average speed $= \dfrac{145}{4} \approx 36$ m/min

Q3c *Average rate of change of distance from home*

$= \dfrac{35-0}{4} \approx 9$ m/min

Q4a $75 - x$, $45 - y$

Q4b



Q4c $C = 21x + 24y + 16(75 - x) + 19(45 - y)$

$C = 5x + 5y + 2055$

Q4d *Lowest* $C = 5\times 35 + 5\times 0 + 2055 = 2230$ dollars

Module 4: Business-related mathematics

Q1a Let $\$x$ be the amount before the increase.

$1.09x = 375.00$, $x \approx 344.04$

Q1b *The increase* $= 375.00 - 344.04 = 30.96$ dollars

Q2a *Monthly payment* $= \dfrac{1}{12}\left(650000 \times \dfrac{4.80}{100} \times 1\right) = 2600$ dollars

Q2b The principal remains the same, $650000.

Q2c Effective rate $= \dfrac{2\times 12}{12+1} \times 4.80\% \approx 8.86\%$

Q3a By calculator.

TVM Solver: N = 360, I% = 6.7, PV = 450 000,

PMT = ? FV = 0, P/Y = 12, C/Y = 12

Monthly repayment ≈ 2903.75 dollars

Q3b *New rate* $= 6.7 - 0.18 = 6.52\%$

TVM Solver: N = ?, I% = 6.52, PV = 446 808.15,

PMT = 2903.75 FV = 0, P/Y = 12, C/Y = 12

$N = 333.69$, .: shorten by $360 - 8 - 333.69 = 18.31 \approx 18$ months

Q3c *Interest saved* $= 2903.75 \times 18.31 \approx 53000$ dollars

Q4a *Annual percentage rise in the cost of living*

$= \dfrac{180.4 - 179.4}{179.4} \times 100\% \approx 0.6\%$

Q4b 0.6%

Q4c Let x be the missing entry.

$\dfrac{x}{100.7} = \dfrac{180.4}{179.5}$, $x \approx 101.2$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors