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# ***Mathematical Methods***

## ***2010***

### ***Trial Examination 1***

## Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

### Question 1

Solve the following system of simultaneous linear equations for  $x$  and  $y$ .

2 marks

$$-0.5x + y = -1.5$$

$$2x + y = 1$$

$$x - 2y = 3$$

### Question 2

Given  $f(x) = x^2 - 2x$  and  $g(x) = -\frac{1}{2}f(1-2x) + \frac{3}{2}$ ,

a. express  $g(x)$  in the form  $a(x-b)(x-c)$ , where  $a, b, c \in R$ .

2marks

b. Describe the relation between the graph of the transformed function  $g(x)$  and the graph of the original function  $f(x)$  in terms of dilations, reflections and translations.

2 marks

**Question 3**

Given  $f(x) = 1 + \log_e x$ ,

a. show that  $f(xy) + f\left(\frac{x}{y}\right)$  is a function of  $x$  only.

2 marks

b. Find the solutions to the equation  $f(xy) + f\left(\frac{x}{y}\right) = 0$ ,  $x, y \in R$ .

2 marks

**Question 4**

Find the exact value of  $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} (\cos^2 x - \sin^2 x) dx$ .

3 marks

**Question 5**

Consider  $f(x) = x^2 + 4x - 5$  and  $g(x) = e^x$ .

a. State the domain of  $f(g(x))$ . 1 mark

b. Determine the range of  $f(g(x))$ . 2 marks

c. Solve  $f(g(x)) = 8(e^x - 1)$  for  $x$ . Write the solution(s) in exact form. 2 marks

**Question 6**

Consider  $h: \left[0, \frac{1}{2}\right] \rightarrow \mathbb{R}$ ,  $h(x) = \frac{1}{2} \cos\left(\pi x - \frac{\pi}{2}\right) - 1$ .

a. Determine the range of  $h$ . 2 marks

b. Determine the inverse of  $h$  including its domain. Express your answer in simplest form. 3 marks

c. Solve  $h(x) = -\frac{3}{4}$  for  $x$ . 2 marks

**Question 7**

a. Factorise  $f(x) = 8x^3 - 12x^2 + 6x - 1$  completely.

2 marks

b. Hence or otherwise find a *factorised* anti-derivative of  $f(x)$ .

1 mark

**Question 8**

Water is poured into a container of irregular shape to a depth of 1.20 metres in 2 minutes at 25 litres per minute. Let the volume of water in the container be  $V$  and the depth of water be  $h$  at time  $t$ .

a. Find  $\frac{dV}{dt}$  in  $\text{cm}^3$  per minute.

1 mark

b. Find  $\frac{\Delta h}{\Delta t}$  in metres per minute.

1 mark

c. Find  $\frac{\Delta h}{\Delta V}$  in  $\text{cm}^{-2}$ .

2 marks

**Question 9**

Two fair dice are rolled together three times and the sum of the two uppermost numbers in each roll is recorded.

- a. Complete the following probability distribution of the random variable  $X$ , the sum of the two uppermost numbers in rolling two fair dice together.

2 marks

$X$	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X = x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$		$\frac{1}{6}$					

- b. Let random variable  $Y$  be the number of times that  $X = 7$  when the two fair dice are rolled together three times. Find  $\Pr(Y \geq 1)$ .

2 marks

**Question 10**

A train arrives at a station at 7:30 am, and departs the station between 7:35 am and 7:40 am. If you arrive at the station *after* 7:33 am, determine the probability that you catch the train.

(Hint: Construct a probability density function which describes the distribution of probability in missing the train if you arrive at the station after time  $t$ .)

4 marks

**End of exam 1**