itute.com

= Year 12 **= Calculus II** **= Worksheet 10**

1. The rate of change of V with respect to t is inversely proportional to $t+1$. Initially $V=100$ and $V=80$ when $t=5$. Set up a differential equation for V, solve it to find V when $t=9$.

2. A population grows at a rate proportional to its size. If the initial population is 10000 and it doubles every unit of time. Find the population after (i) 2 (ii) 3 (iii) 2.73 units of time.

3. The rate of decay of a radioactive substance is directly proportional to the remaining mass m of the substance. The time taken for a half of the substance remaining in the sample is 3.2 hours. Find the proportion of the substance remaining in the sample after *another* two hours.

4. The gradient of the tangent to a curve $y=f(x)$ is partly proportional to x and partly to $\frac{1}{\sqrt{x}}$. The curve passes through the origin, $(1,2)$ and $(4,11)$. Find y when $x=9$.

5. The surface temperature T of an object changes in time t at a rate proportional to the difference between the temperature of the object and the temperature T_o of the surrounding medium. If the temperature of the object drops by 10°C in 5 minutes. Find the drop in temperature in the next 5 minutes, given the surrounding temperature is constant 20°C and the initial temperature is 80°C.

6. The acceleration a of a particle moving in a straight line is directly proportional to the square of its speed v. It has an initial speed of 80 ms^{-1}. Five seconds later the speed is 56 ms^{-1}. Find the time when the speed is 10 ms^{-1}.

7. A thermometer is taken from a house at 21°C to the outside. One minute later it reads 27°C, another minute later it reads 30°C. Find the temperature outside the house.

8. A person borrows \$10000 at 10.95% interest compounded daily. Set up a differential equation for the amount owing at time t days. Find the amount \$$A$ owing a year later.

9. A tank contains 2000 L of salt solution with a concentration of 0.3 kg of salt per litre. Pure water runs into the tank at 50 L per minute and the well mixed solution runs out at the same rate. Find the amount of salt in the tank after 5 minutes.

10. Refer to Q9. Instead of pure water, a solution with a concentration of 0.2 kg of salt per litre runs into the tank. Find the amount of salt in the tank after 5 minutes. Find the concentration of salt in the tank eventually.

11. Refer to Q9. Instead of running out at the same rate, the well mixed solution runs out at 40 L per minute. Use Euler's method (step size of 1 minute) to find the approximate amount of salt in the tank after 5 minutes.

Numerical, algebraic and worded answers.

1. $dV/dt = k/t$, ≈ 74.3
2. (i) 40000 (ii) 80000 (iii) $\approx$ 66346
3. 0.3242
4. 45
5. 8.3°C
6. 81.7s
7. 33°C
8. $dA/dt = (\log_e 1.0003)A$ \$11157.02
9. 529.5 kg
10. 576.5 kg, 0.2 kg per litre
11. 542.9 kg